Guest Lecture

CSE 100

Professor Martin Tompa from the Computer Science and Engineering Department tells us about ...

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Today's topic

Secret Codes,

Unforgeable Signatures,

and

Coin Flipping on the Phone

FIT 100 A Secret Code

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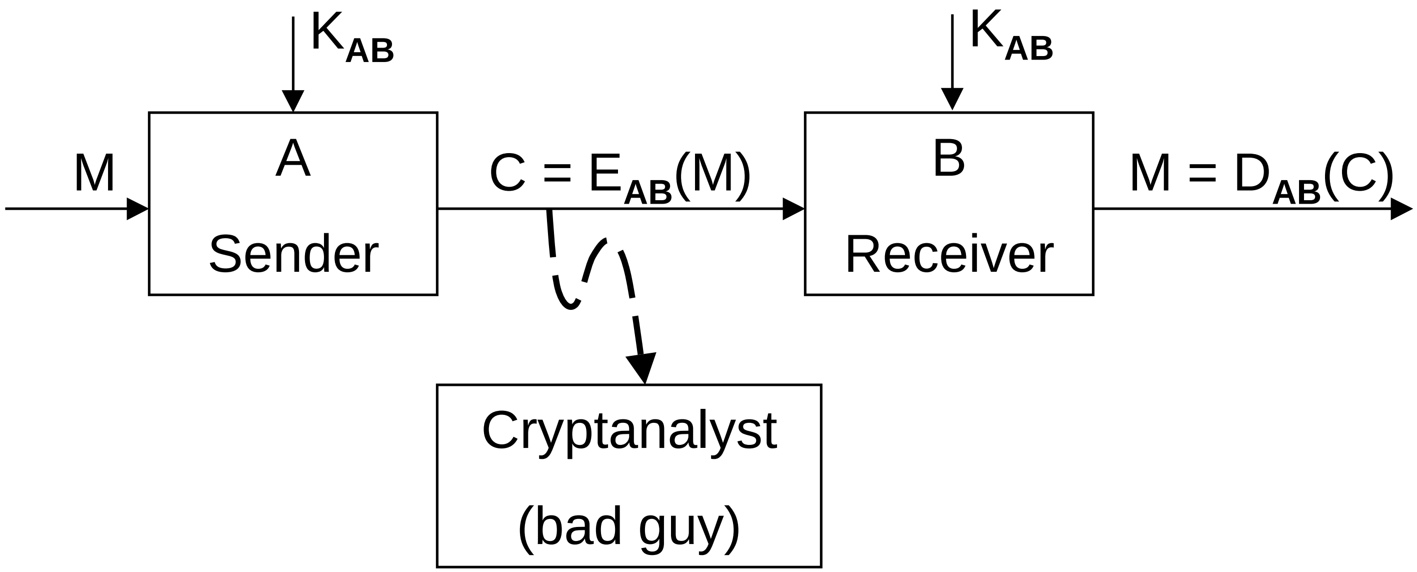
What Is a Cryptosystem?



M	C	K_{AB}
Message	Encryption	Key
Plaintext	Cyphertext	
Cleartext		

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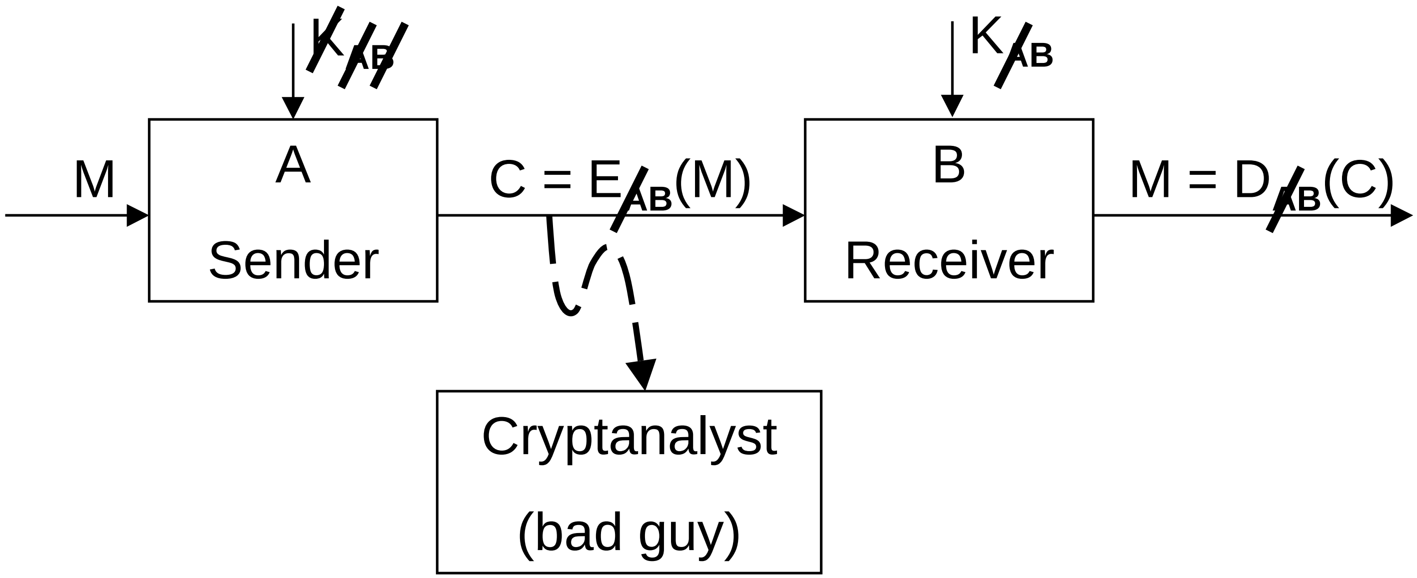
What Is a Public Key Cryptosystem?



M	C	K_B	E_B
Message	Encryption	Key	Public Key
Plaintext	Cyphertext	Private Key	
Cleartext			

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The RSA Public Key Cryptosystem

- Invented by Rivest, Shamir, and Adleman in 1977.
- Has proven resilient to all cryptanalytic attacks since.

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Receiver's Set-Up

- Choose 500-digit primes p and q (each 2 more than a multiple of 3).

  $p = 5$, $q = 11$

- Let $n = pq$.

  $n = 55$

- Let $s = (1/3)\ (2(p - 1)(q - 1) + 1)$.

  $s = (1/3)\ (2 \cdot 4 \cdot 10 + 1) = 27$

- Publish n.

  Keep p, q and s secret.

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Encrypting a Message

- Break the message into chunks.

  H I   C H R I S …

- Translate each chunk into an integer M ($0 < M < n$).

  8 9   3 8 18 9 19 …

- Divide M^3 by n. $E(M)$ is the remainder.

  $M = 8$, $n = 55$

  $8^3 = 512 = 9 \times 55 + 17$

  $E(8) = 17$

FIT 100 Decrypting A Cyphertext C

- Divide C^s by n. $D(C)$ is the remainder.

  $C = 17$, $n = 55$, $s = 27$

  $17^{27} = 1{,}667{,}711{,}322{,}168{,}688{,}287{,}513{,}535{,}727{,}415{,}473$

  $= 30{,}322{,}024{,}039{,}430{,}696{,}136{,}609{,}740{,}498{,}463 \times 55 + 8$

  $D(17) = 8$

- Translate $D(C)$ into letters.

  H

FIT 100 Why does it work?

Euler's Theorem (1736): Suppose

- p and q are distinct primes,
- $n = pq,$
- $0 \leq M < n,$ and
- $k > 0.$

If $M^{k(p-1)(q-1)+1}$ is divided by n, the remainder is M.

$$(M^3)^s = (M^3)^{(1/3)(2(p-1)(q-1)+1)}$$

$$= M^{2(p-1)(q-1)+1}$$

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Leonhard Euler 1707-1783

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Why Is It Secure?

- To find *M = D(C),* you seem to need *s.*
- To find s, you seem to need *p* and *q.*
- All you have is *n = pq.*
- How hard is it to factor a 1000-digit number *n*?

  With the grade school method,

  doing 10,000,000 steps per second

  it would take … 10^{485} years.

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State of the Art in Factoring

- 1977: Inventors encrypt a challenge using “RSA129,” a 129-digit number $n = pq$.
- 1981: Pomerance invents a new factoring method.
- 1994: RSA129 factored over an 8 month period using 1000 computers on the Internet around the world.

- With this method, a 250-digit number would take 100,000,000 times as long.

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Signed Messages

- How A sends a secret message to B

<u>A</u> <u>B</u>

$C = E_{\boldsymbol{B}}(M)$

C →

$M = D_{\boldsymbol{B}}(C)$

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Signed Messages

- How A sends a secret message to B

A		B
$C = E_{\boldsymbol{B}}(M)$		
	C →	
		$M = D_{\boldsymbol{B}}(C)$

- How A sends a signed message to B

A		B
$C = D_{\boldsymbol{A}}(M)$		
	C →	
		$M = E_{\boldsymbol{A}}(C)$

FIT 100 Signed *and* Secret Messages

- How A sends a secret message to B ...

A	B

$C = E_{\boldsymbol{B}}(M)$

C →

$M = D_{\boldsymbol{B}}(C)$

- How A sends a signed secret message to B ...

A	B

$C = E_{\boldsymbol{B}}(D_{\boldsymbol{A}}(M))$

C →

$M = E_{\boldsymbol{A}}(D_{\boldsymbol{B}}(C))$

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Flipping a Coin Over the Phone

A		B
Choose random x.		
$y = E_{\mathbf{A}}(x)$		
	y →	
		Guess if x is even or odd.
	← "even" / "odd"	
	x →	
		Check $y = E_{\mathbf{A}}(x)$.

❖ B wins if the guess about x was right