# Warm Up

Translate this sentence into symbolic logic, and describe a weather pattern and transportation method that causes the proposition to be false.

It is snowing today, and if it is raining or snowing then we won't walk to school.

# Warm Up – Solution

Translate this sentence into symbolic logic, and describe a weather pattern and transportation method that causes the proposition to be false.

It is snowing today, and if it is raining or snowing then we won’t walk to school.

Robbie’s process: identify connecting words, identify propositions, figure out parentheses.

# Warm Up – Solution

Translate this sentence into symbolic logic, and describe a weather pattern and transportation method that causes the proposition to be false.

It is snowing today, and if it is raining or snowing then we won’t walk to school.

Identify connecting words: look for and, or, not, if-then, etc.

# Warm Up – Solution

Translate this sentence into symbolic logic, and describe a weather pattern and transportation method that causes the proposition to be false.

$p$ $q$ $p$ $\neg r$

It is snowing today, and if it is raining or snowing then we won't walk to school.

Identify propositions: What's left are propositions, look for repeats and hidden negations.

$p$:it is snowing today.
$q$: it is raining.
$r$: we walk to school.

# Warm Up – Solution

Translate this sentence into symbolic logic, and describe a weather pattern and transportation method that causes the proposition to be false.

$p$ $q$ $p$ $\neg r$

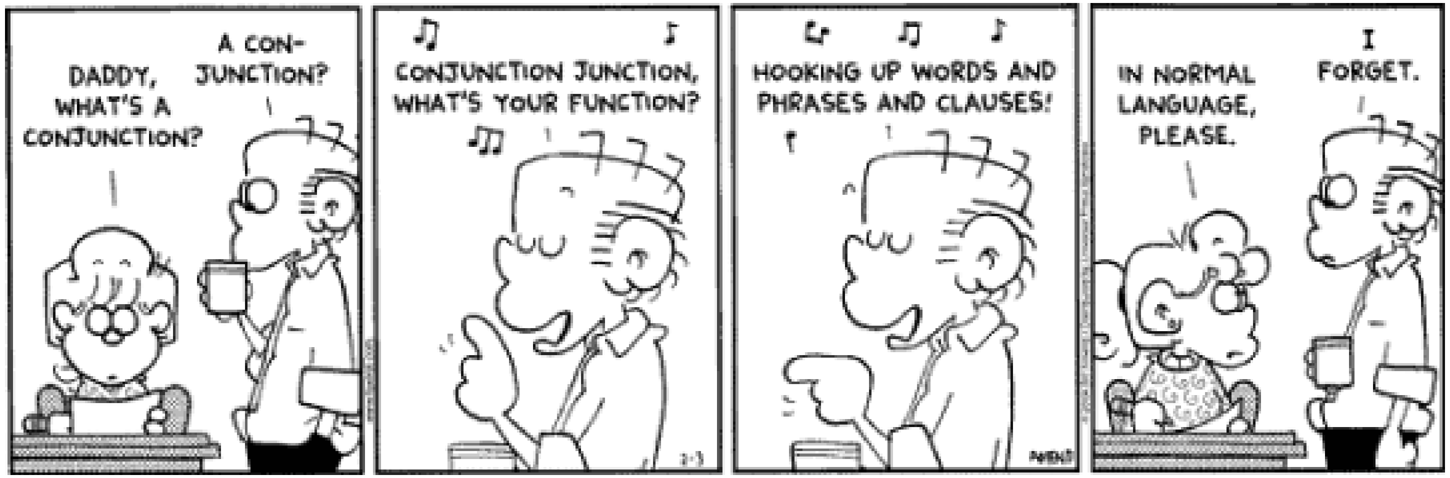
It is snowing today, and $\boxed{\text{if}}$ it is raining or snowing $\boxed{\text{then}}$ we won't walk to school.

$p$:it is snowing today.
$q$: it is raining.
$r$: we walk to school.

Figure out parentheses

$$p \wedge [(q \vee p) \rightarrow \neg r]$$

"raining or snowing" is the condition of the implication, not walking to school is the conclusion. Omitted words in other clauses hint that "It is snowing today" stands on its own.




# More Logic, Equivalences, Symbolic Proofs

CSE 311 Fall 2020
Lecture 2

# Announcements

Lecture recordings on “panopto” – link is posted on ed.

Section walkthroughs will go there as well.

Homework 1 comes out this afternoon (hooray!) – on the calendar on the webpage

Due (a-week-from-today) Friday night.

You’ll submit to gradescope (more information coming).

# CSE 390Z

**CSE 390Z is a workshop** designed to provide academic support to students enrolled concurrently in CSE 311. During each 1.5-hour workshop, students will reinforce concepts through:

- **collaborative problem solving**
- **practice study skills and effective learning habits**
- **build community for peer support**

All students enrolled in CSE 311 are welcome to register for this class.If you are interested in receiving an add code, please fill out a form here. If you have any questions or concerns please contact Rob (minneker@uw.edu).

# Today

Syllabus – Homework Policies

Simplification Rules

Our first proof!

# Collaboration Policy

The goal of the course is for you to learn; the collaboration policy facilitates that.

Collaboration with others is **encouraged**

- You may work on homework with other students, but…

BUT you must:

- list anyone you work with
- Write up your solutions independently using your own words.

Some details

- do not leave with any solution written down or photographed
- wait 30 minutes before writing up your solution

If you can write the solution yourself, you've **taught** each other, and you've **learned!**

And that makes it much more likely to stick.

More implementation details (including sample scenarios) on the webpage!

# Late Work

You have 4 late days for the quarter.

You can use a late day to turn in a homework up to 24 hours later than otherwise due.

But you can use at most 2 late days per assignment.

The intent of late days is to handle "normal" issues that happen during a quarter (e.g. programming project for another course takes longer than you thought, or a family member had a birthday ~~party~~zoom you had to attend).

If you have a more significant issue, please talk to us. We'll work with you.

- Private post on Ed, or email Robbie

It's a weird quarter. We can work with you when things happen, but we're not psychics. Please reach out – the sooner the better.

- If we give you an extension and it turns out you didn't need it, that's ok!

# More Connectives

# A More Complicated Statement

"Robbie knows the Pythagorean Theorem if he is a mathematician and took geometry, and he is a mathematician or did not take geometry."

Is this a proposition?

We'd like to *understand* what this proposition means.

In particular, is it true?

# A Compound Proposition

"Robbie knows the Pythagorean Theorem if he is a mathematician and took geometry, and he is a mathematician or did not take geometry."

We'd like to *understand* what this proposition means.

First find the simplest (atomic) propositions:

| | |
|---|---|
| $p$ | "Robbie knows the Pythagorean Theorem" |
| $q$ | "Robbie is a mathematician" |
| $r$ | "Robbie took geometry" |

($p$ if ($q$ and $r$)) and ($q$ or (not $r$))

$$(p \text{ if } (q \wedge r)) \wedge (q \vee (\neg r))$$

# Parentheses…

"Robbie knows the Pythagorean Theorem if he is a mathematician and took geometry, and he is a mathematician or did not take geometry."

$$(p \text{ if } (q \wedge r)) \wedge (q \vee (\neg r))$$

| | |
|---|---|
| $p$ | **"Robbie knows the Pythagorean Theorem"** |
| $q$ | **"Robbie is a mathematician"** |
| $r$ | **"Robbie took geometry"** |

How did we know where to put the parentheses?

- Subtle English grammar choices (top-level parentheses are independent clauses).
- Context/which parsing will make more sense.
- Conventions

A reading on this is coming soon!

# Back to the Compound Proposition…

"Robbie knows the Pythagorean Theorem if he is a mathematician and took geometry, and he is a mathematician or did not take geometry."

$$(p \text{ if } (q \wedge r)) \wedge (q \vee (\neg r))$$

| | |
|---|---|
| $p$ | **"Robbie knows the Pythagorean Theorem"** |
| $q$ | **"Robbie is a mathematician"** |
| $r$ | **"Robbie took geometry"** |

What promise am I making?

$$(\,(q \wedge r) \rightarrow p) \wedge (q \vee (\neg r)) \qquad\qquad (p \rightarrow (q \wedge r)) \wedge (q \vee (\neg r))$$

The first one! Being a mathematician and taking geometry is the condition. Knowing the Pythagorean Theorem is the promise.

# Analyzing the Sentence with a Truth Table

| $p$ | $q$ | $r$ | $\neg r$ | $q \vee \neg r$ | $q \wedge r$ | $(q \wedge r) \rightarrow p$ | $((q \wedge r) \rightarrow p) \wedge (q \vee \neg r)$ |
|---|---|---|---|---|---|---|---|
| F | F | F | T | T | F | T | T |
| F | F | T | F | F | F | T | F |
| F | T | F | T | T | F | T | T |
| F | T | T | F | T | T | F | F |
| T | F | F | T | T | F | T | T |
| T | F | T | F | F | F | T | F |
| T | T | F | T | T | F | T | T |
| T | T | T | F | T | T | T | T |

# Order of Operations

Just like you were taught PEMDAS

e.g. $3 + 2 \cdot 4 = 11$ not $24$.

Logic also has order of operations.

Parentheses

Negation

And

Or

Implication

Biconditional

For this course: each of these is it's own level!
e.g. "and"s have precedence over "or"s

Within a level, apply from left to right.

Other authors place And, Or at the same level – it's good practice to use parentheses even if not required.

# Logical Connectives

| | |
|---|---|
| **Negation (not)** | $\neg p$ |
| **Conjunction (and)** | $p \land q$ |
| **Disjunction (or)** | $p \lor q$ |
| **Exclusive Or** | $p \oplus q$ |
| **Implication(if-then)** | $p \longrightarrow q$ |
| **Biconditional** | $p \leftrightarrow q$ |

These ideas have been around for so long everything has at least two names – even “and.”

Two more connectives to discuss!

# Biconditional: $p \leftrightarrow q$

Think: $(p \rightarrow q) \land (q \rightarrow p)$

$p$ if and only if $q$

$p$ *iff* $q$

$p$ is equivalent to $q$

$p$ implies $q$ and $q$ implies $p$

$p$ is necessary and sufficient for $q$

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

# Biconditional: $p \leftrightarrow q$

Think: $(p \rightarrow q) \land (q \rightarrow p)$

*p* if and only if *q*

*p iff q*

*p* is equivalent to *q*

*p* implies *q* and *q* implies *p*

*p* is necessary and sufficient for *q*

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| **T** | **T** | **T** |
| **T** | **F** | **F** |
| **F** | **T** | **F** |
| **F** | **F** | **T** |

$p \leftrightarrow q$ is the proposition: "$p$" and "$q$" have the same truth value.

# Exclusive Or

Exactly one of the two is true.

$p \oplus q$

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

In English "either $p$ or $q$" is the most common, but be careful.

Often "$p$ or $q$" where you're just supposed to understand that exclusive or is meant.

Try to say "either…or…" in your own writing.

# Exclusive Or

Exactly one of the two is true.

$p \oplus q$

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

In English "either $p$ or $q$" is the most common, but be careful.

Often translated "$p$ or $q$" where you're just supposed to understand that exclusive or (nor the "regular" inclusive or " $\vee$ ") is meant.

Try to say "either…or…" in your own writing.

# Today

A proof!

We want to be able to make iron-clad guarantees that something is true.

And convince others that we really have ironclad guarantees.

But first, some notation.

# Logical Equivalence

We will want to talk about whether two propositions are "the same."

Two propositions are "equal" ($=$) if they are character-for-character identical.

$p \wedge q = p \wedge q$ but $p \wedge q \neq q \wedge p$

We almost never ask whether propositions are equal. It's not an interesting question.

Two propositions are "equivalent" ($\equiv$) if they always have the same truth value.

$p \wedge q \equiv p \wedge q$ and $p \wedge q \equiv q \wedge p$

But $p \wedge q \not\equiv p \vee q$
When $p$ is true and $q$ is false: $p \wedge q$ is false, but $p \vee q$ is true.

# $A \leftrightarrow B$ vs. $A \equiv B$

$A \equiv B$ is an ***assertion over all possible truth values*** that $A$ and $B$ always have the same truth values.

Use $A \equiv B$ when you're manipulating propositions ("doing algebra")

$A \leftrightarrow B$ is a ***proposition*** that may be true or false depending on the truth values of the variables in $A$ and $B$.

$A \equiv B$ and $(A \leftrightarrow B) \equiv \mathbf{T}$ have the same meaning.

# Simplification and Proofs

# Manipulating Expressions

When we're doing algebra, we can apply rules to transform expressions

$(a+b)(c+d) = ac + ad + bc + bd$ or $ab + ac = a(b+c)$

We want rules for logical expressions too.

For each rule, we'll:

1. Derive it/make sure we understand why it's true.
2. Practice using it.

By the end of the course, you'll do these "automatically" on full sentences; for now we'll practice mechanically on symbolic forms.

As you're practicing, don't lose sight of the *intuition* for what you're doing.

# De Morgan’s Laws

Negate the statement

“my code compiles or there is a bug.”

i.e. find a natural English sentence that says $\neg(p \vee q)$

“the following is not true: my code compiles or there is a bug”

Hint: when it the original sentence false?

‘or’ means ‘at least one is true’ so to negate, we need to say ‘neither is true’ or equivalently ‘both are false’

$\neg p \wedge \neg q$

“my code does not compile and there is not a bug”

# De Morgan's Laws

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

is a general rule. It's always true for any propositions $p$ and $q$.
This is one of De Morgan's Laws.

The other is:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

# De Morgan's Laws

Example: $\neg(p \wedge q) \equiv \neg p \vee \neg q$

| $p$ | $q$ | $\neg p$ | $\neg q$ | $\neg p \vee \neg q$ | $p \wedge q$ | $\neg(p \wedge q)$ | $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$ |
|---|---|---|---|---|---|---|---|
| T | T | F | F | F | T | F | T |
| T | F | F | T | T | F | T | T |
| F | T | T | F | T | F | T | T |
| F | F | T | T | T | F | T | T |

# De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

```
if (!(front != null && value > front.data))
      front = new ListNode(value, front);
else {
      ListNode current = front;
      while (current.next != null && current.next.data < value))
            current = current.next;
      current.next = new ListNode(value, current.next);
}
```

# De Morgan's Laws

$$\neg(p \land q) \equiv \neg p \lor \neg q$$
$$\neg(p \lor q) \equiv \neg p \land \neg q$$

```
!(front != null && value > front.data)
```

$$\equiv$$

```
front == null || value <= front.data
```

**You've been using these for a while!**

# Law of Implication

Activity

Implications are not totally intuitive.
AND/OR/NOT make more intuitive sense to me…
can we rewrite implications using just ANDs ORs and NOTs?

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

One approach: think "when is this implication false?" then negate it (you might want one of DeMorgan's Laws!

# Law of Implication

Implications are hard.
AND/OR/NOT make more intuitive sense to me…
can we rewrite implications using just ANDs ORs and NOTs?

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Seems like we might want $\neg(p \wedge \neg q)$

$\neg p \vee q$

Seems like a reasonable guess.
So is it true? Is $\neg p \vee q \equiv p \rightarrow q$?

# Law of Implication

$$\neg p \lor q \equiv p \to q$$

| $\boldsymbol{p}$ | $\boldsymbol{q}$ | $\boldsymbol{p \to q}$ | $\boldsymbol{\neg p}$ | $\boldsymbol{\neg p \lor q}$ | $\boldsymbol{(\neg p \lor q) \leftrightarrow (p \to q)}$ |
|---|---|---|---|---|---|
| T | T | T | F | T | T |
| T | F | F | F | F | T |
| F | T | T | T | T | T |
| F | F | T | T | T | T |

# Properties of Logical Connectives

We've derived two facts about logical connectives.

There's a lot more. A LOT more.

The next slide is a list of a bunch of them…

- Most of these are much less complicated than the last two, so we won't go through them in detail.
- DO NOT freak out about how many there are. We will always provide you the list on the next slide (no need to memorize).

# Properties of Logical Connectives

**We will always give you this list!**

For every propositions $p, q, r$ the following hold:

- **Identity**
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- **Domination**
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- **Idempotent**
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- **Commutative**
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- **Associative**
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- **Absorption**
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- **Negation**
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$

# Using Our Rules

WOW that was a lot of rules.

Why do we need them? Simplification!

Let's go back to the "law of implication" example.

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

When is the implication true? Just "or" each of the three "true" lines!

$$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$$

Also seems pretty reasonable

So is $(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv (\neg p \vee q)$

i.e. are these both alternative representations of $p \to q$?

# Our First Proof

We could make another truth table (you should! It's a good exercise)

But we have another technique that is nicer.

Let's try that one

Then talk about why it's another good option.

We're going to give an iron-clad guarantee that:

$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv \neg p \vee q$

i.e. that this is another valid "law of implication"

# Our First Proof

How do we write a proof?

It’s not always plug-and-chug…we’ll be highlighting strategies throughout the quarter.

To start with:

Make sure we know what we want to show…

# Our First Proof

$$(p \land q) \lor (\neg p \land q) \lor (\neg p \land \neg q) \equiv$$

None of the rules look like this

Practice of Proof-Writing:
**Big Picture**…WHY do we think this might be true?

The last two "pieces" came from the vacuous proof lines…maybe the "$\neg p$" came from there? Maybe that **simplifies** down to $\neg p$

$$\equiv (\neg p \lor q)$$

# Let’s apply a rule

$(\neg p \wedge q) \vee (\neg p \wedge \neg q)$

The law says:

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$(\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv \neg p \wedge (q \vee \neg q)$

# Our First Proof

$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv$

None of the rules look like this

Practice of Proof-Writing:
**Big Picture**...WHY do we think this might be true?

The last two "pieces" came from the vacuous proof lines...maybe the "$\neg p$" came from there? Maybe that **simplifies** down to $\neg p$

$\equiv (\neg p \vee q)$

# Our First Proof

$$(p \land q) \lor (\neg p \land q) \lor (\neg p \land \neg q) \equiv (p \land q) \lor [(\neg p \land q) \lor (\neg p \land \neg q)]$$

Set ourselves an intermediate goal.
Let's try to simplify those last two pieces

**Associative law**
Connect up the things we're working on.

$$\equiv (\neg p \lor q)$$

# Our First Proof

$$
\begin{aligned}
(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \\
&\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)]
\end{aligned}
$$

Set ourselves an intermediate goal.
Let's try to simplify those last two pieces

Distributive law
We think $\neg p$ is important, let's isolate it.

$$\equiv (\neg p \vee q)$$

# Our First Proof

$$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)]$$
$$\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)]$$
$$\equiv (p \wedge q) \vee [\neg p \wedge \mathrm{T}]$$

Set ourselves an intermediate goal.
Let's try to simplify those last two pieces

**Negation**
Should make things simpler.

$$\equiv (\neg p \vee q)$$

# Our First Proof

$$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)]$$
$$\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)]$$
$$\equiv (p \wedge q) \vee [\neg p \wedge \mathrm{T}]$$
$$\equiv (p \wedge q) \vee [\neg p]$$

Set ourselves an intermediate goal.
Let’s try to simplify those last two pieces

**Identity**
Should make things simpler.

$$\equiv (\neg p \vee q)$$

# Our First Proof

$$
\begin{aligned}
(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \\
&\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)] \\
&\equiv (p \wedge q) \vee [\neg p \wedge \mathrm{T}] \\
&\equiv (p \wedge q) \vee [\neg p] \\
&\equiv [\neg p] \vee (p \wedge q)
\end{aligned}
$$

**Stay on target:**
We met our intermediate goal.
Don't forget the final goal!
We want to end up at $(\neg p \vee q)$

If we apply the distribution rule,
We'd get a $(\neg p \vee q)$

$$\equiv (\neg p \vee q)$$

# Our First Proof

$$(p \land q) \lor (\neg p \land q) \lor (\neg p \land \neg q) \equiv (p \land q) \lor [(\neg p \land q) \lor (\neg p \land \neg q)]$$
$$\equiv (p \land q) \lor [\neg p \land (q \lor \neg q)]$$
$$\equiv (p \land q) \lor [\neg p \land \mathrm{T}]$$
$$\equiv (p \land q) \lor [\neg p]$$
$$\equiv [\neg p] \lor (p \land q)$$

**Stay on target:**
We met our intermediate goal.
Don't forget the final goal!
We want to end up at $(\neg p \lor q)$

If we apply the distribution rule,
We'd get a $(\neg p \lor q)$

$$\equiv (\neg p \lor q)$$

**Commutative**
Make the expression look exactly like the law (more on this later)

# Our First Proof

$$
\begin{aligned}
(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \\
&\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)] \\
&\equiv (p \wedge q) \vee [\neg p \wedge \mathrm{T}] \\
&\equiv (p \wedge q) \vee [\neg p] \\
&\equiv [\neg p] \vee (p \wedge q) \\
&\equiv (\neg p \vee p) \wedge (\neg p \vee q)
\end{aligned}
$$

**Stay on target:**
We met our intermediate goal.
Don't forget the final goal!
We want to end up at $(\neg p \vee q)$

If we apply the distribution rule,
We'd get a $(\neg p \vee q)$

**Distributive** $\equiv (\neg p \vee q)$
Creates the $(\neg p \vee q)$ we were hoping for.

# Our First Proof

$$
\begin{aligned}
(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \\
&\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)] \\
&\equiv (p \wedge q) \vee [\neg p \wedge \mathrm{T}] \\
&\equiv (p \wedge q) \vee [\neg p] \\
&\equiv [\neg p] \vee (p \wedge q) \\
&\equiv (\neg p \vee p) \wedge (\neg p \vee q) \\
&\equiv (p \vee \neg p) \wedge (\neg p \vee q) \\
&\equiv \mathrm{T} \wedge (\neg p \vee q) \\
&\equiv (\neg p \vee q)
\end{aligned}
$$

**Stay on target:**
We met our intermediate goal.
Don't forget the final goal!
We want to end up at $(\neg p \vee q)$

If we apply the distribution rule,
We'd get a $(\neg p \vee q)$

**Commutative**
Make the expression look exactly like the law (more on this later)

**Identity**
Simplifies the part we want to disappear.

# Our First Proof

$$
\begin{aligned}
(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \\
&\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)] \\
&\equiv (p \wedge q) \vee [\neg p \wedge \mathrm{T}] \\
&\equiv (p \wedge q) \vee [\neg p] \\
&\equiv [\neg p] \vee (p \wedge q) \\
&\equiv (\neg p \vee p) \wedge (\neg p \vee q) \\
&\equiv (p \vee \neg p) \wedge (\neg p \vee q) \\
&\equiv \mathrm{T} \wedge (\neg p \vee q) \\
&\equiv (\neg p \vee q) \wedge \mathrm{T} \\
&\equiv (\neg p \vee q)
\end{aligned}
$$

**Stay on target:**
We met our intermediate goal.
Don't forget the final goal!
We want to end up at $(\neg p \vee q)$

If we apply the distribution rule,
We'd get a $(\neg p \vee q)$

**Commutative** followed by **Identity**
Look exactly like the law, then apply it.

We're done!!!

# Commutativity

We had the expression $(p \wedge q) \vee [\neg p]$

But before we applied the distributive law, we switched the order…why?

The law says $p \vee (q \wedge r) \equiv (\mathrm{p} \vee q) \wedge (p \vee r)$

not $(q \wedge r) \vee p \equiv (q \vee p) \wedge (r \vee p)$

So **technically** we needed to commute first.

Eventually (in about 2 weeks) we’ll skip this step. For now, we’re doing two separate steps.

Remember this is the “training wheel” stage. The point is to be careful.

# More on Our First Proof

We now have an ironclad guarantee that

$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv (\neg p \vee q)$

Hooray! But we could have just made a truth-table. Why a proof?

Here’s one reason.

Proofs don’t *just* give us an ironclad guarantee. They’re also an explanation of *why* the claim is true.

The key insight to our simplification was “the last two pieces were the vacuous truth parts – the parts where $p$ was false”

That’s in there, *in the proof*.

# Our First Proof

$$
\begin{aligned}
(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] && \text{Associative} \\
&\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)] && \text{Distributive} \\
&\equiv (p \wedge q) \vee [\neg p \wedge \mathrm{T}] && \text{Negation} \\
&\equiv (p \wedge q) \vee [\neg p] && \text{Identity} \\
&\equiv [\neg p] \vee (p \wedge q) && \text{Commutative} \\
&\equiv (\neg p \vee p) \wedge (\neg p \vee q) && \text{Distributive} \\
&\equiv (p \vee \neg p) \wedge (\neg p \vee q) && \text{Commutative} \\
&\equiv \mathrm{T} \wedge (\neg p \vee q) && \text{Negation} \\
&\equiv (\neg p \vee q) \wedge \mathrm{T} && \text{Commutative} \\
&\equiv (\neg p \vee q) && \text{Identity}
\end{aligned}
$$

The last two terms are "vacuous truth" maybe the simplify to $\neg p$

$p$ no longer matters in $p \wedge q$ if $\neg p$ automatically makes the expression true.

# More on Our First Proof

With practice (and quite a bit of squinting) you can see not just the ironclad guarantee, but also the reason why something is true.

That's not easy with a truth table.

Proofs can also communicate intuition about why a statement is true.

We'll practice extracting intuition from proofs more this quarter.