CSE 311: Foundations of Computing I

Homework 2 (due Friday, October 15th at 11:00 PM)

Directions: *Write up carefully argued solutions to the following problems. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. However, you may use results from lecture, the theorems handout, and previous homeworks without proof.*

For proofs in Propositional Logic, below, you must cite every rule that you apply, including *Commutativity* and *Associativity*. You may apply only a single rule per line. However, you can apply that rule to multiple parts of the formula as long as they are *non-overlapping*.

1. Say Anything (0 points)

Approximately how much time (in minutes) did you spend on each problem of this homework? Were any problems especially difficult or especially interesting? This information is useful for us to calibrate the difficulty of the problems for future offerings of this course.

2. The Equalizer (20 points)

Prove the following assertions using a series of logical equivalences.

(Feel free to use this web site to complete the problem. After creating an account with your **UW email**, click on "CSE 311 HW2" to create a new project that is preloaded with a file for each of the parts below. Once you have solved each part, take a screenshot of your completed solution and include it in your submission.)

(a) [6 Points] $(p \to r) \to p \equiv p$.

(b) [6 Points] $p \to (r \to p) \equiv \mathsf{T}$.

(c) [2 Points] We saw in lecture that "$\wedge$" and "$\vee$" are associative. Is "$\to$" also associative?
Hint: it may be useful to cite (a) and (b) above.

(d) [6 Points] $(p \to r) \wedge (\neg s \to \neg p) \equiv p \to (r \wedge s)$.

3. Tangled (20 points)

(a) [2 Points] Translate the Boolean Algebra expression $(X' + XY') + (Z' + ZY)$ to Propositional Logic. Use the variables q, r, and s to represent the propositions $X = 1$, $Y = 1$, and $Z = 1$, respectively.

(b) [16 Points] Prove that your solution to (a) is a tautology using a chain of equivalences.

Hint: The Distributivity rule will likely be useful here.

(Feel free to use this tool to check that your solution is correct. Take a screenshot of your completed solution in the tool and include it in your submission rather than writing it out by hand.)

(c) [2 Points] Why do we know that the Boolean Algebra expression from part (a) is always 1? Explain.

4. Two For the Road (16 points)

In lecture 4, we constructed a circuit to compute the number of classes (lectures or sections) remaining on a given day of the week. That circuit had four outputs (c_0, c_1, c_2, c_3) to indicate 0, 1, 2, or 3. In this problem, we construct a circuit that has only two outputs, $(s_1 s_0)_2$, that give the same output but as a *binary* number.

(a) [6 Points] Write a table showing the values for s_1 and s_0 in each possible case of the input values (d_2, d_1, d_0, L).

(b) [5 Points] Write s_1 in either product-of-sums form or sum-of-products form. (Your choice.)

(c) [5 Points] Write s_0 in whichever form (product-of-sums or sum-of-products) you did **not** use in part (b).

5. Lost In Translation (16 points)

Let the domain of discourse be plants and animals. Let's define the predicates $\mathsf{Plant}(x)$ and $\mathsf{Animal}(x)$ to mean that x is a plant or an animal, respectively. Define the predicates $\mathsf{Eats}(x, y)$ and $\mathsf{HasWings}(x)$ to mean that x eats y and x has wings, respectively.

Translate each of the following logical statements into English. You should not simplify. However, you should use the techniques shown in lecture for producing more natural translations when restricting domains and for avoiding the introduction of variable names when not necessary.

(a) [4 Points] $\neg\exists x\, (\mathsf{Plant}(x) \land \mathsf{HasWings}(x))$

(b) [4 Points] $\exists x\, \big(\mathsf{Plant}(x) \land \exists y\, (\mathsf{Animal}(y) \land \neg\, \mathsf{Eats}(x, y) \land \neg\, \mathsf{Eats}(y, x))\big)$

(c) [4 Points] $\exists x\, \big(\mathsf{Plant}(x) \land \exists y\, (\mathsf{Animal}(y) \land \mathsf{Eats}(x, y) \land \mathsf{HasWings}(y))\big)$

(d) [4 Points] $\forall x\, \big(\mathsf{Animal}(x) \to \exists y\, ((\mathsf{Plant}(y) \lor \mathsf{Animal}(y)) \land \mathsf{Eats}(x, y))\big)$

6. Mystic Pizza (16 points)

Let the domain of discourse be pizzas and toppings. We define the predicates $\mathsf{Pizza}(x)$ and $\mathsf{Topping}(x)$ to mean that x is a type of pizza or a topping, respectively. We also define the predicate $\mathsf{Ingredient}(x, y)$ to mean that x is a type of pizza that has y as a topping. You can also assume an "$=$" operator that is true when x and y are the same types of pizzas or the same toppings.

Translate each of the following English statements into predicate logic. Do not simplify.

(a) [4 Points] No pizza has all the toppings.

(b) [4 Points] Two pizzas have a topping in common.

(c) [4 Points] Some pizza has only one topping.

(d) [4 Points] Two toppings are not *both* used on any pizza.

7. She's All That (12 points)

The questions below consider the two propositions

$$\forall x\,(P(x) \vee Q(x)) \quad \text{and} \quad (\forall x\, P(x)) \vee (\forall x\, Q(x))$$

where P and Q are some predicates.

(a) [6 Points] Give examples of predicates P and Q and a domain of discourse so that the two propositions **do not** have the same truth value.

(b) [6 Points] Give examples of predicates P and Q and a domain of discourse where the two propositions **do** have the same truth value.

(c) [0 Points] **Extra credit**: What logical relationship always holds between the two propositions? Explain.

8. Extra Credit: Hook (0 points)

Five pirates, called Ann, Brenda, Carla, Danielle and Emily, found a treasure of 100 gold coins. On their ship, they decide to split the coins using the following scheme:

- The first pirate in alphabetical order becomes the chief pirate.
- The chief proposes how to share the coins, and all other pirates (excluding the chief) vote for or against it.
- If 50% or more of the pirates vote for it, then the coins will be shared that way.
- Otherwise, the chief will be thrown overboard, and the process is repeated with the pirates that remain.

Thus, in the first round Ann is the chief: if her proposal is rejected, she is thrown overboard and Brenda becomes the chief, etc; if Ann, Brenda, Carla, and Danielle are thrown overboard, then Emily becomes the chief and keeps the entire treasure.

The pirates' first priority is to stay alive: they will act in such a way as to avoid death. If they can stay alive, they want to get as many coins as possible. Finally, they are a blood-thirsty bunch, if a pirate would get the same number of coins if she voted for or against a proposal, she will vote against so that the pirate who proposed the plan will be thrown overboard.

Assuming that all 5 pirates are intelligent (and aware that all the other pirates are just as aware, intelligent, and bloodthirsty), what will happen? Your solution should indicate which pirates die, and how many coins each of the remaining pirates receives.