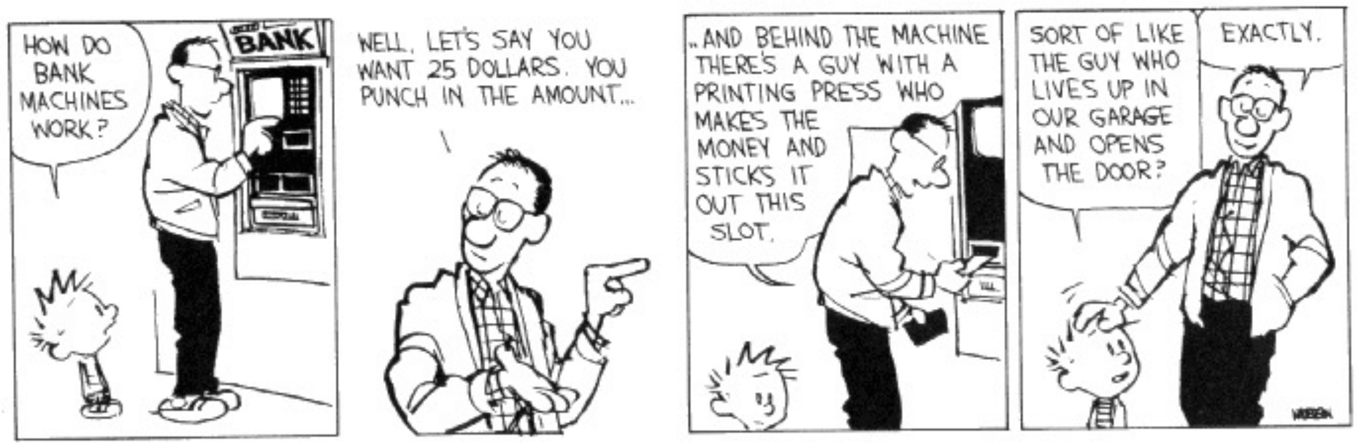
CSE 311: Foundations of Computing

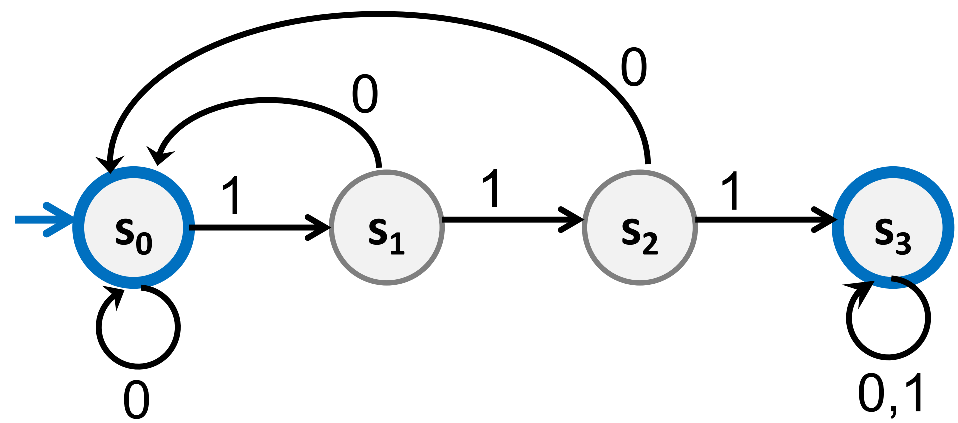
Lecture 24: FSMs with Output and Minimization



Last class: Finite State Machines

- States
- Transitions on input symbols
- Start state and final states
- The “language recognized” by the machine is the set of strings that reach a final state from the start

Old State	0	1
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_0	s_3
s_3	s_3	s_3

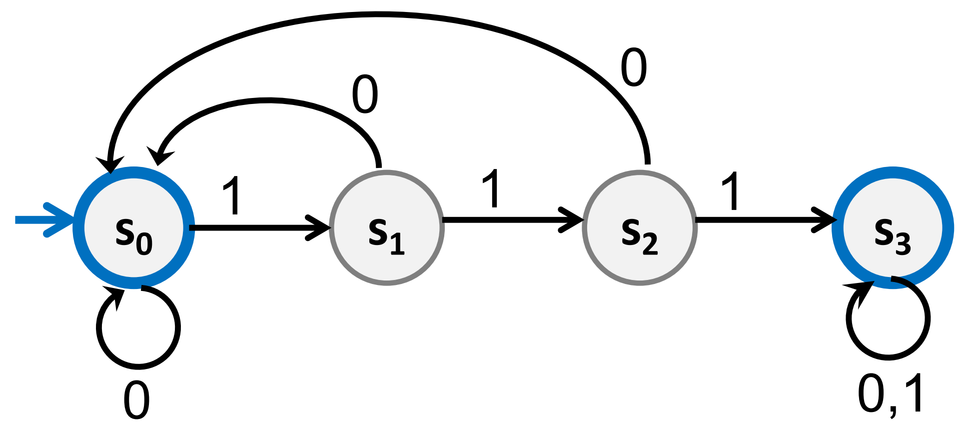


Last class: Finite State Machines

- Each machine designed for strings over some fixed alphabet Σ.

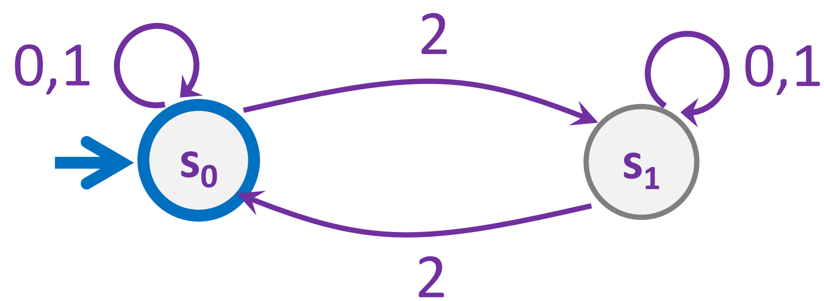
- Must have a transition defined from each state for ***every*** symbol in Σ.

Old State	0	1
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_0	s_3
s_3	s_3	s_3

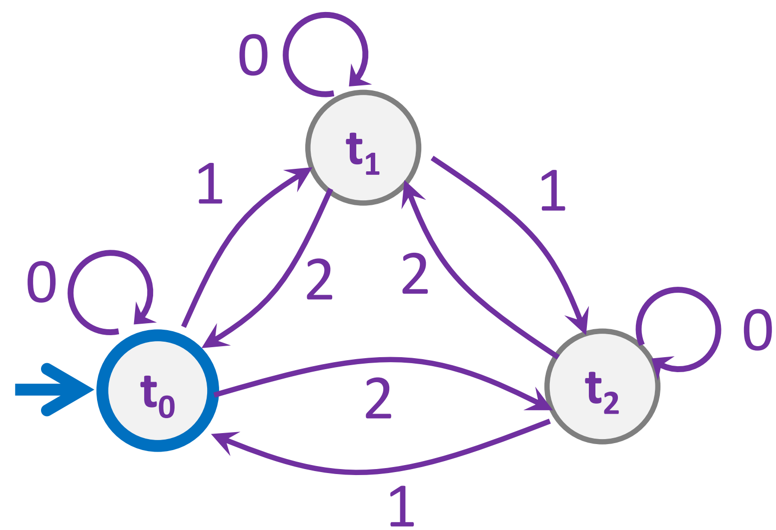


Strings over {0, 1, 2}

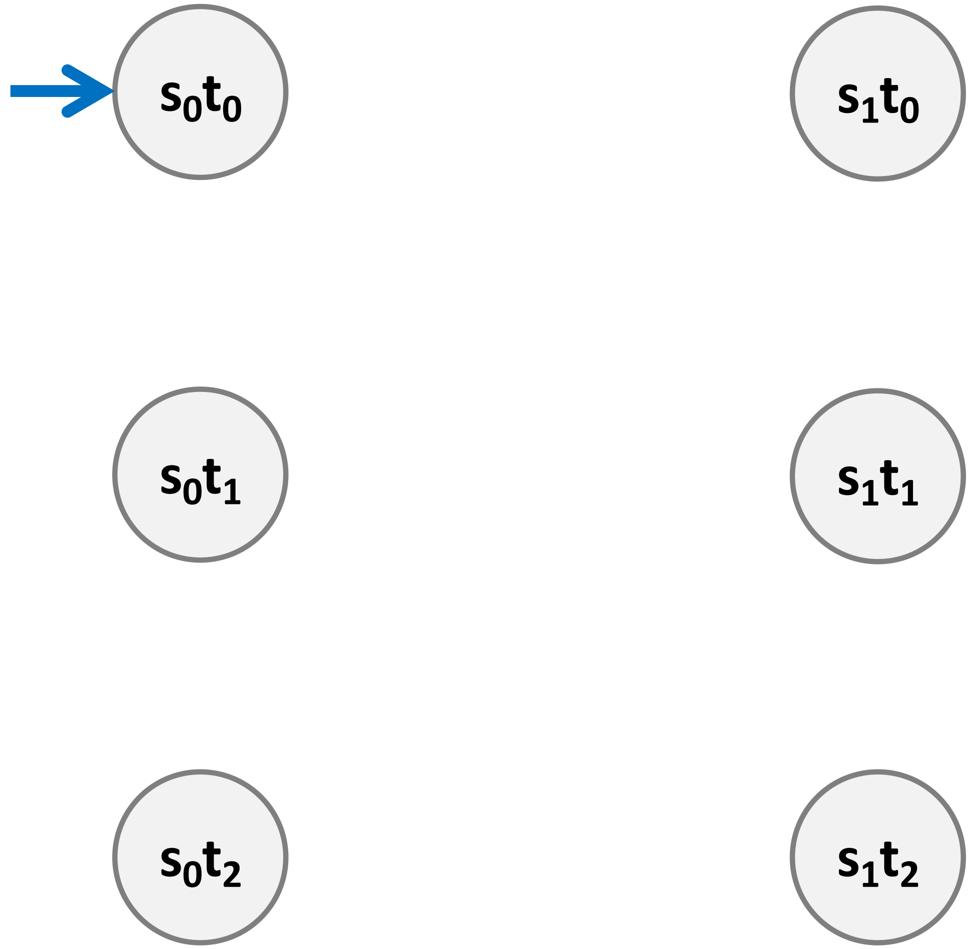
M_1: Strings with an even number of 2's



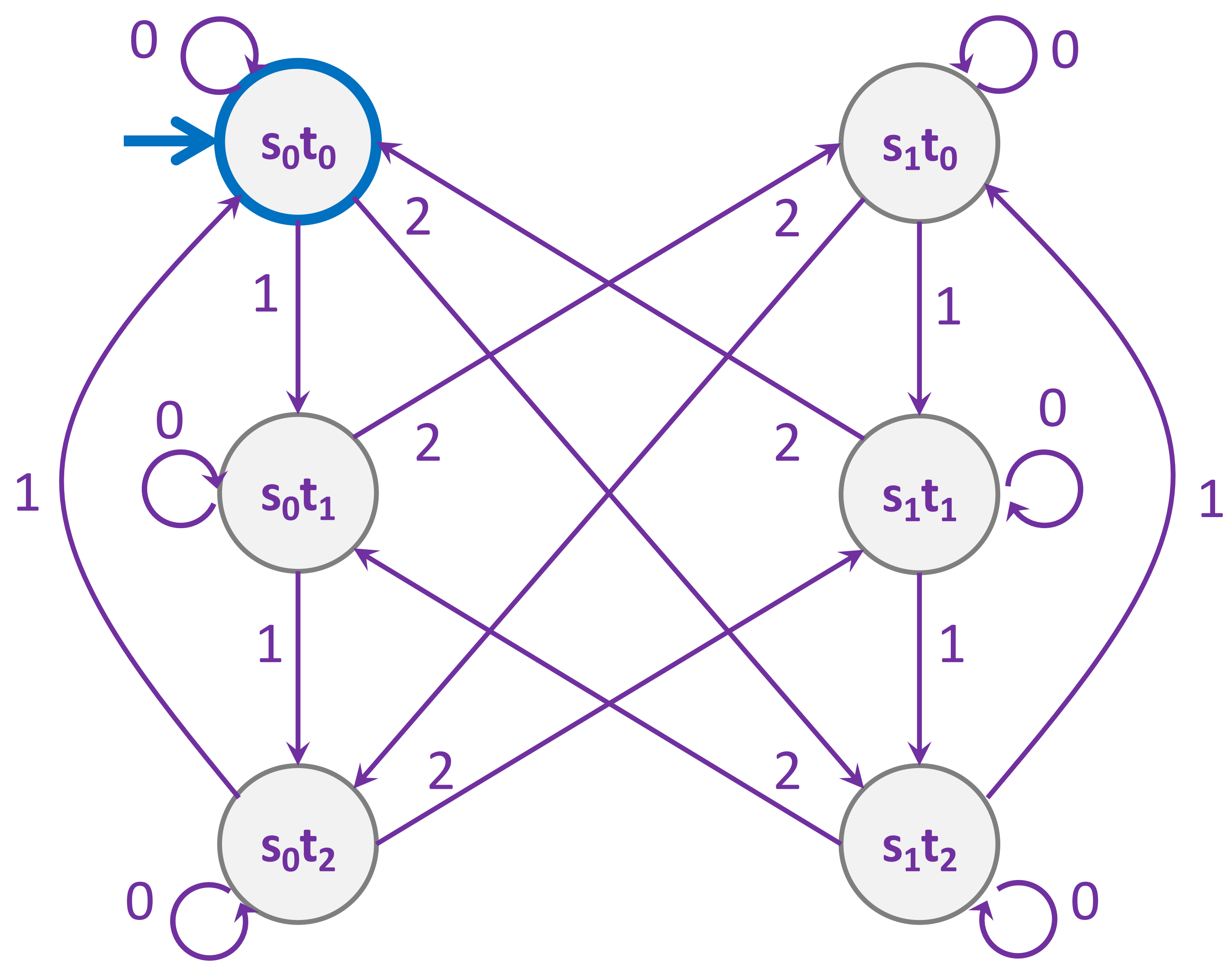
M_2: Strings where the sum of digits mod 3 is 0



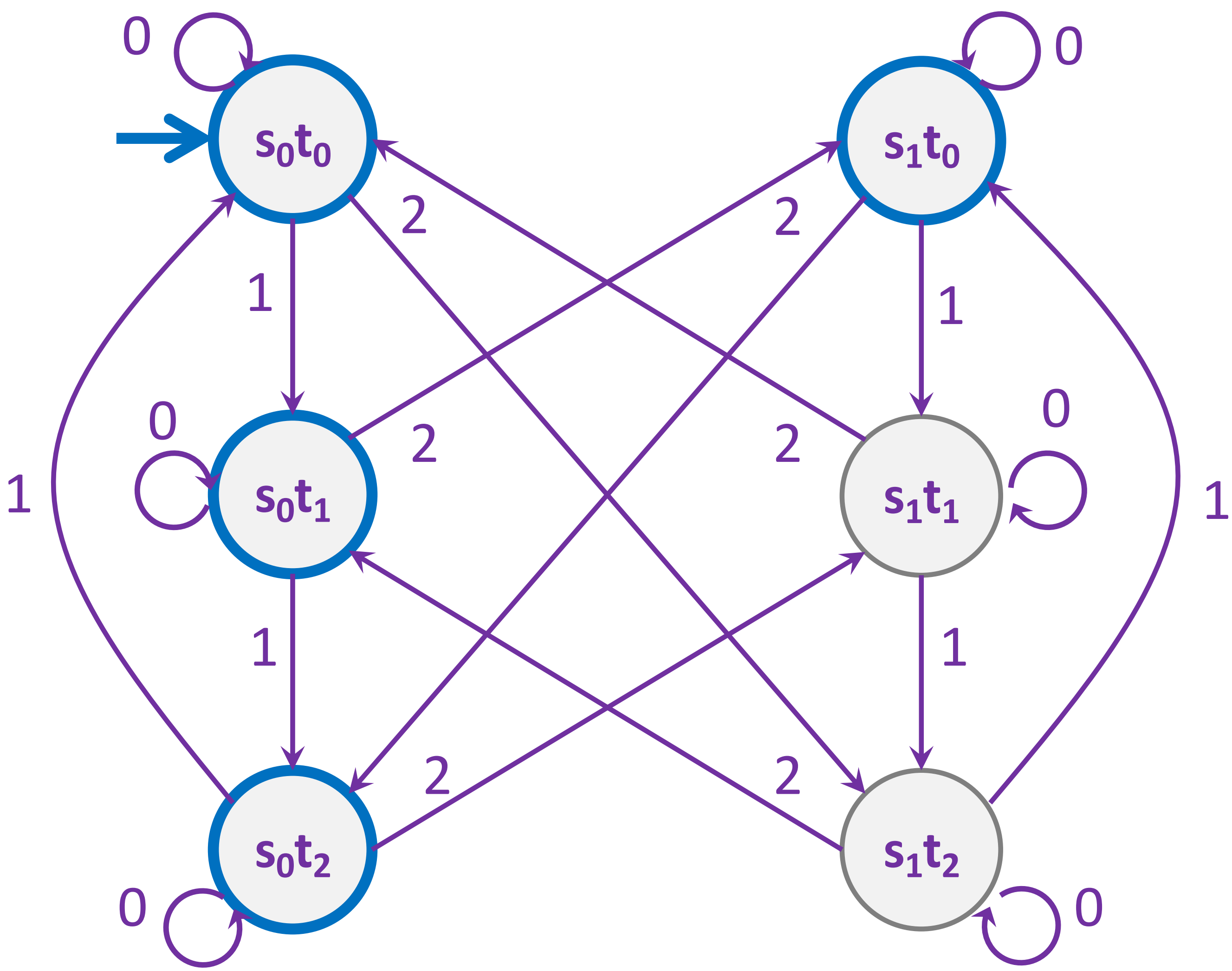
Strings over {0,1,2} w/ even number of 2's and mod 3 sum 0



Strings over {0,1,2} w/ even number of 2's and mod 3 sum 0

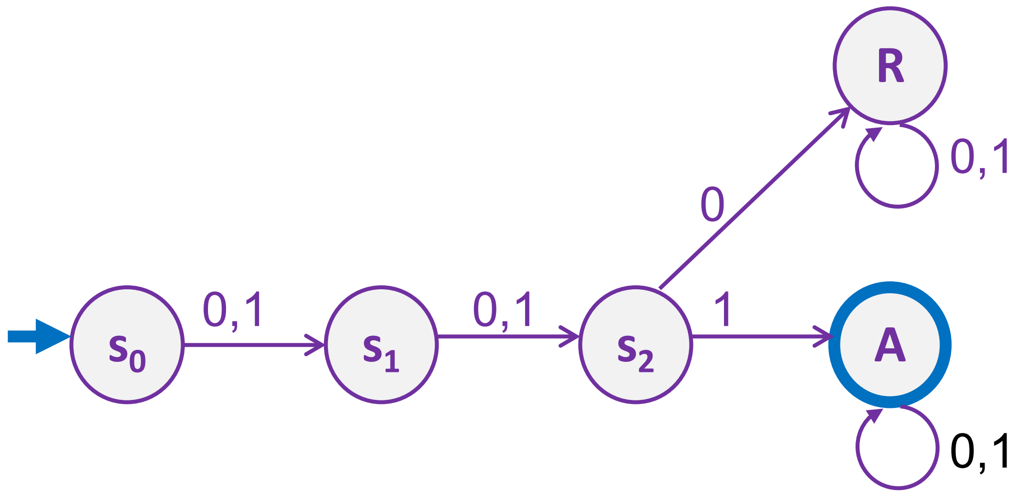


Strings over {0,1,2} w/ even number of 2's OR mod 3 sum 0



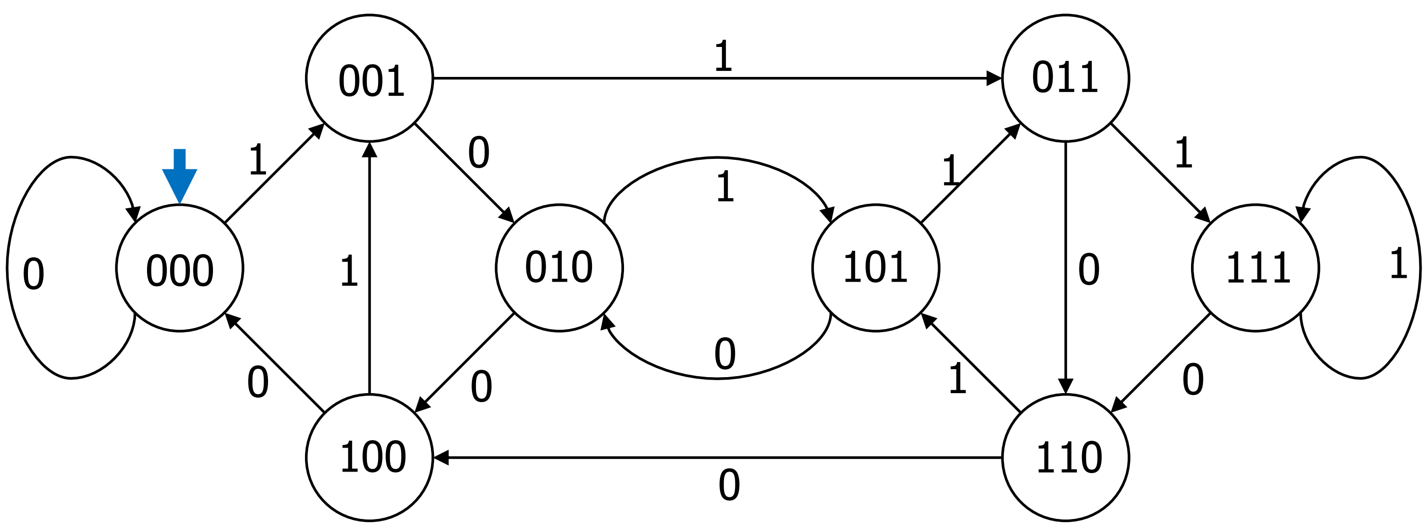
The set of binary strings with a 1 in the 3rd position from the start

The set of binary strings with a 1 in the 3rd position from the start



The set of binary strings with a 1 in the 3rd position from the end

3 bit shift register "Remember the last three bits"



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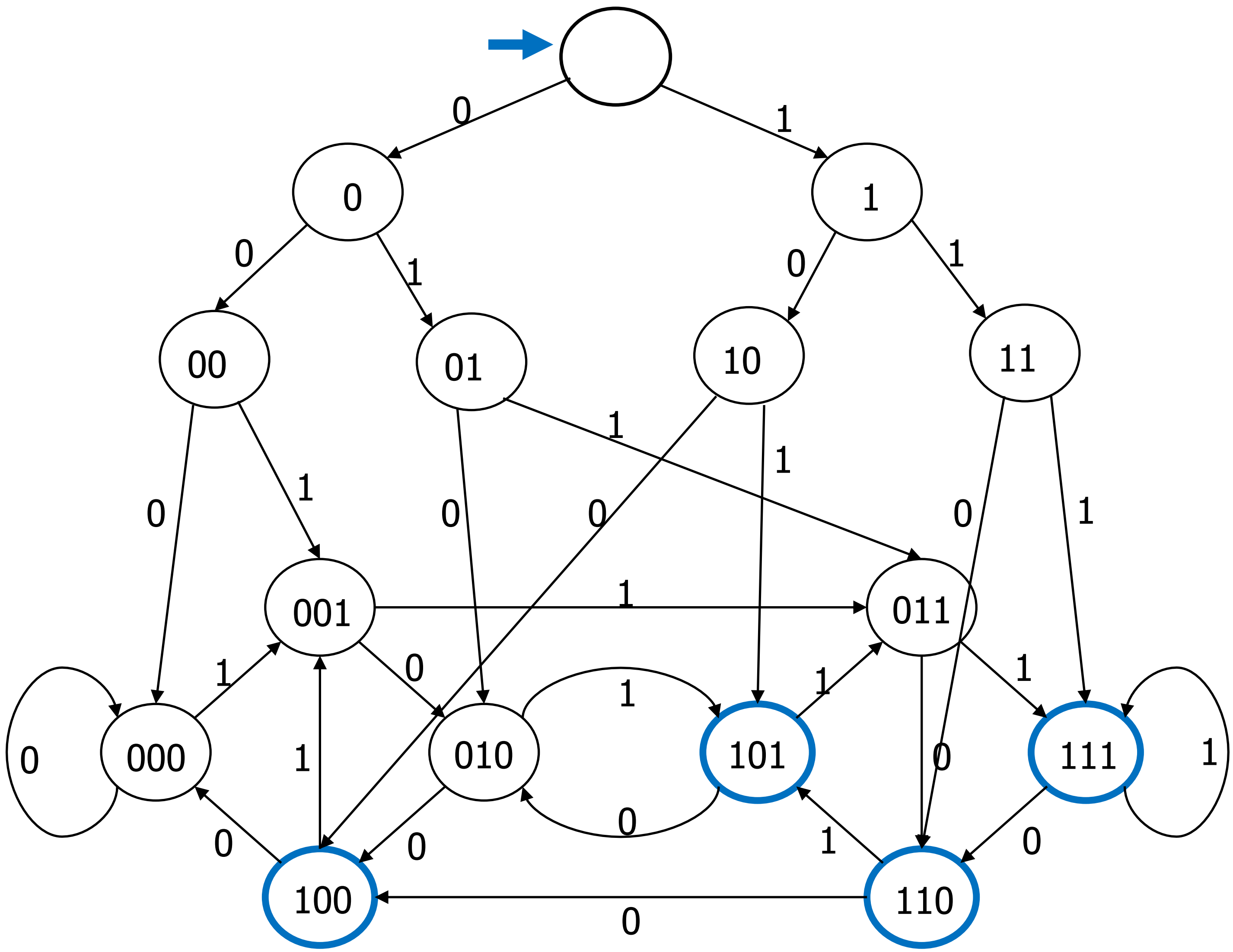

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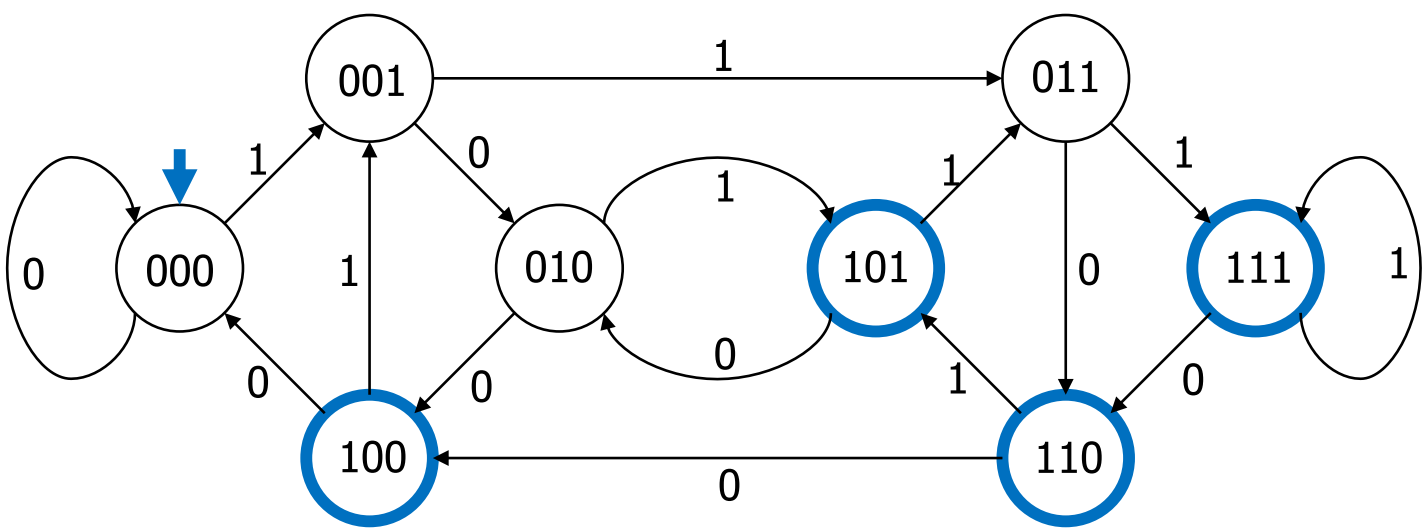
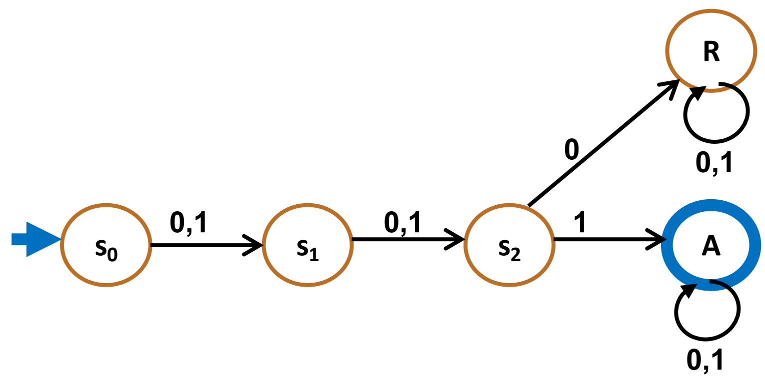
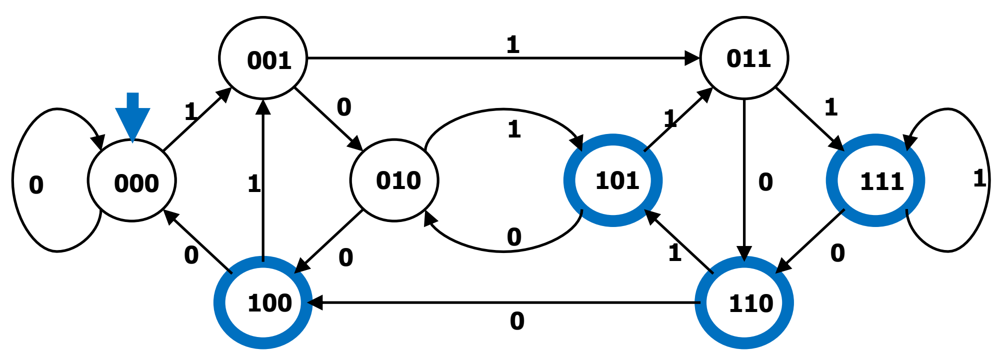


The beginning versus the end





Adding Output to Finite State Machines

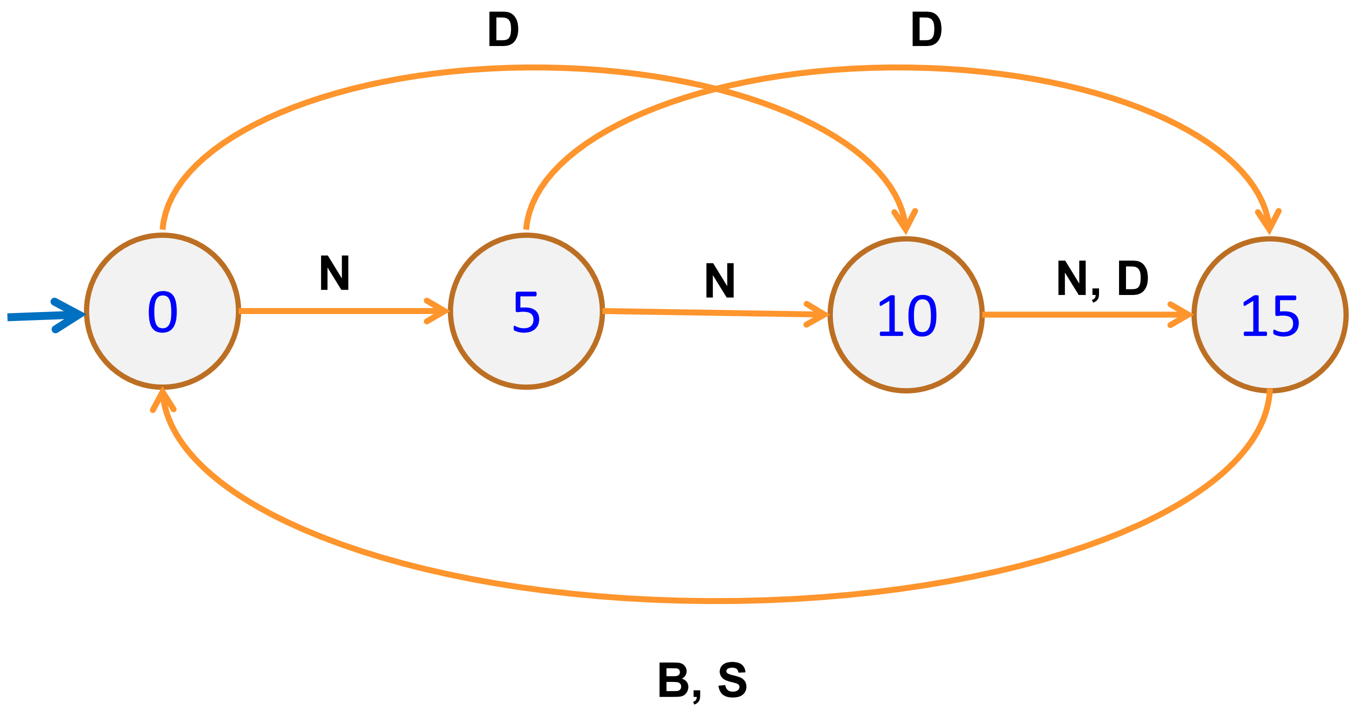
- So far, we have considered finite state machines that just accept/reject strings
  - called "Deterministic Finite Automata" or DFAs

- Now we consider finite state machines *with output*
  - These are the kinds used as controllers

Vending Machine

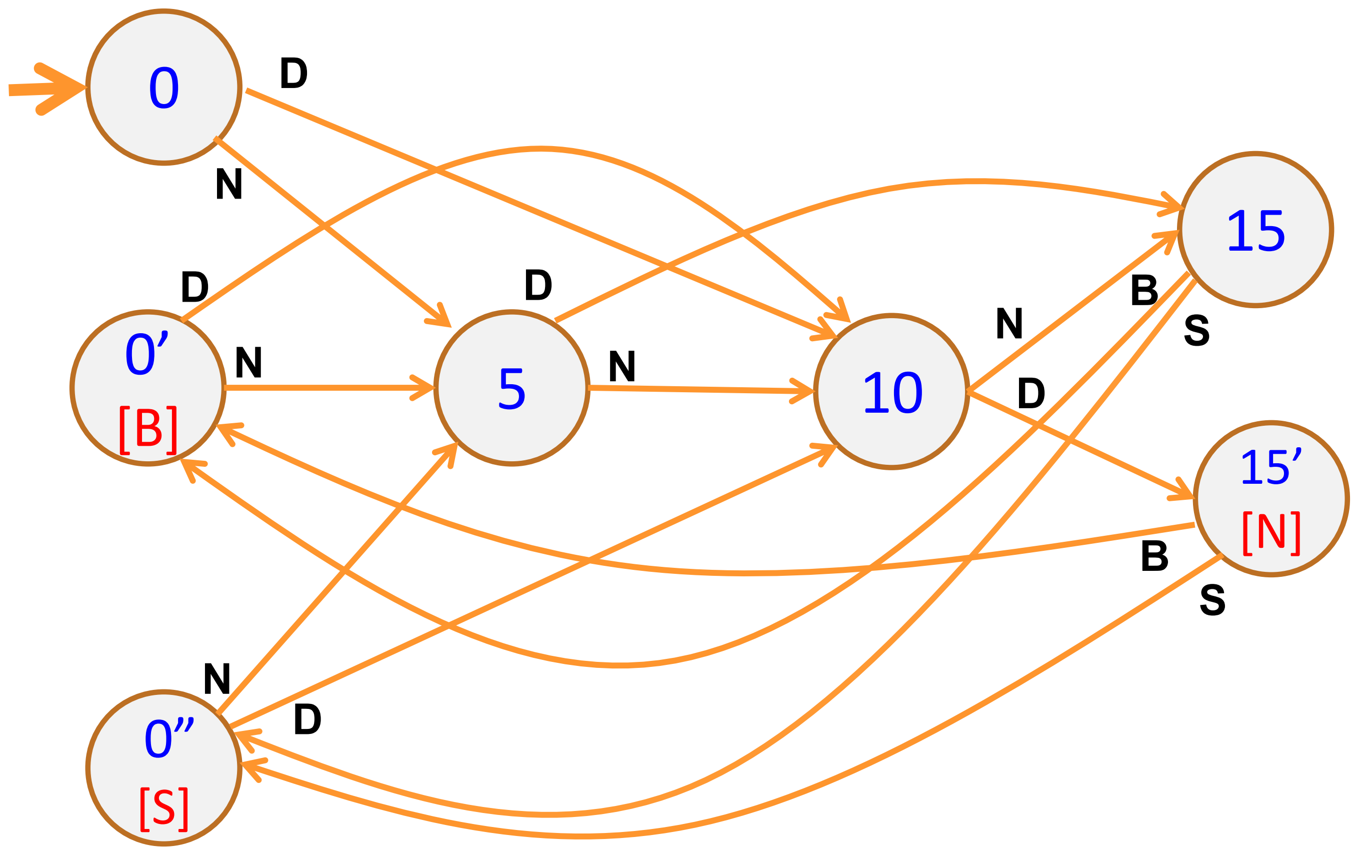
Enter 15 cents in dimes or nickels
Press S or B for a candy bar

Vending Machine, v0.1



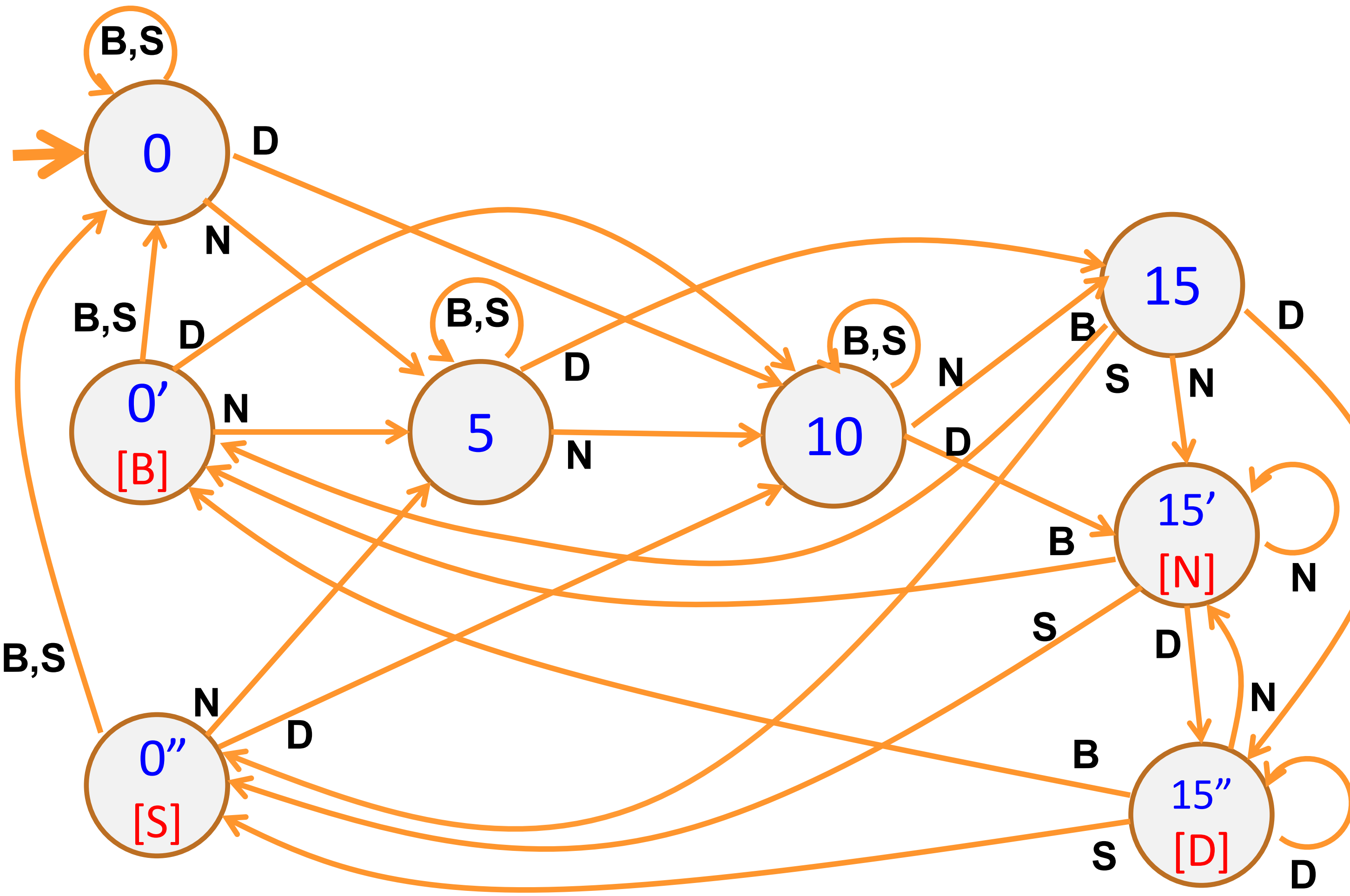
Basic transitions on **N** (nickel), **D** (dime), **B** (butterfinger), **S** (snickers)

Vending Machine, v0.2



Adding output to states: N – Nickel, S – Snickers, B – Butterfinger

Vending Machine, v1.0



Adding additional “unexpected” transitions to cover all symbols for each state

State Minimization

- Many different FSMs (DFAs) for the same problem
- Take a given FSM and try to reduce its state set by combining states
  - Algorithm will always produce the unique minimal equivalent machine (up to renaming of states) but we won't prove this

State Minimization Algorithm

- Put states into groups
- Try to find groups that can be collapsed into one state
  - states can keep track of information that isn't necessary to determine whether to accept or reject
- Group states together until we can *prove* that collapsing them can change the accept/reject result
  - find a specific string x such that:

    starting from state A, following edges according to x ends in accept

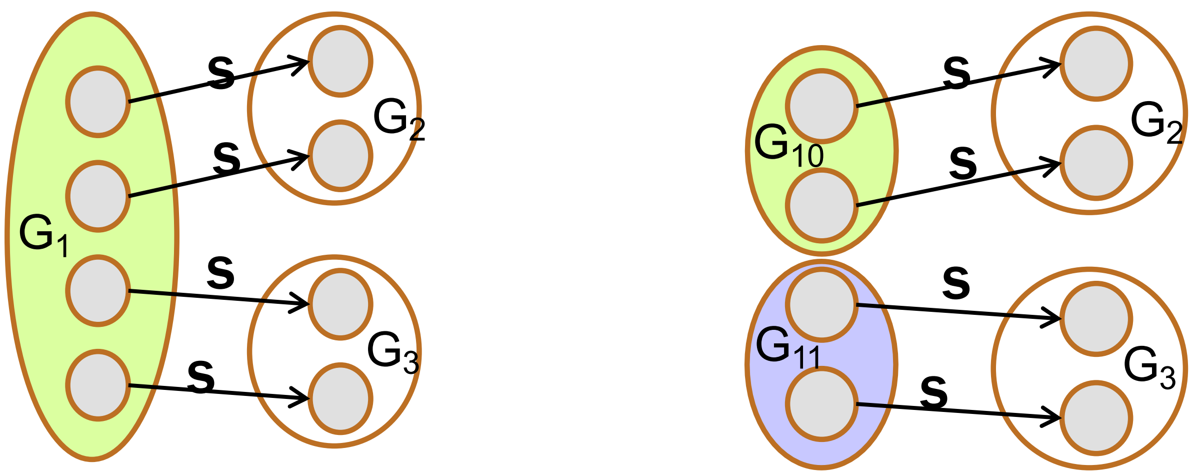
    starting from state B, following edges according to x ends in reject
  - (algorithm below could be modified to show these strings)

State Minimization Algorithm

1. Put states into groups based on their outputs (whether they accept or reject)

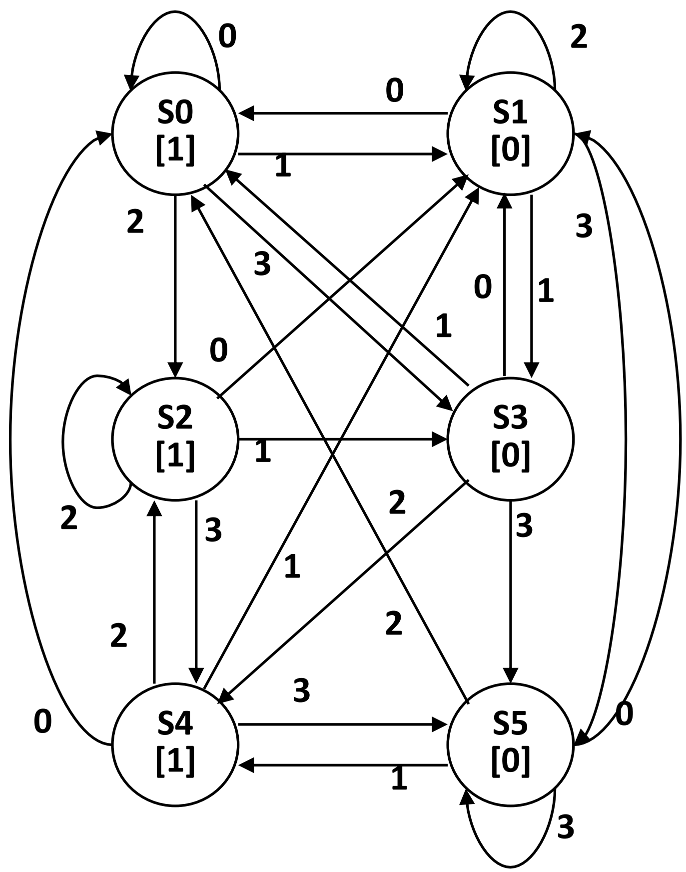
State Minimization Algorithm

1. Put states into groups based on their outputs (whether they accept or reject)
2. Repeat the following until no change happens
   a. If there is a symbol **s** so that not all states in a group **G** agree on which group **s** leads to, split **G** into smaller groups based on which group the states go to on **s**



3. Finally, convert groups to states

State Minimization Example

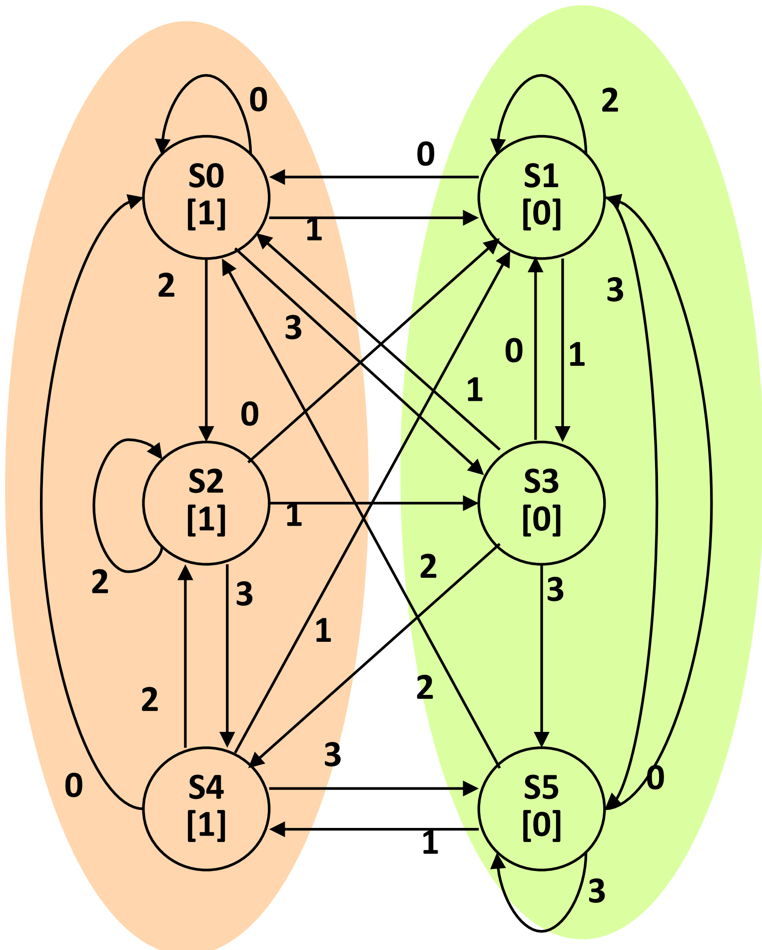


present state	next state 0	1	2	3	output
S0	S0	S1	S2	S3	1
S1	S0	S3	S1	S5	0
S2	S1	S3	S2	S4	1
S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

state
transition table

Put states into groups based on their outputs (or whether they accept or reject)

State Minimization Example



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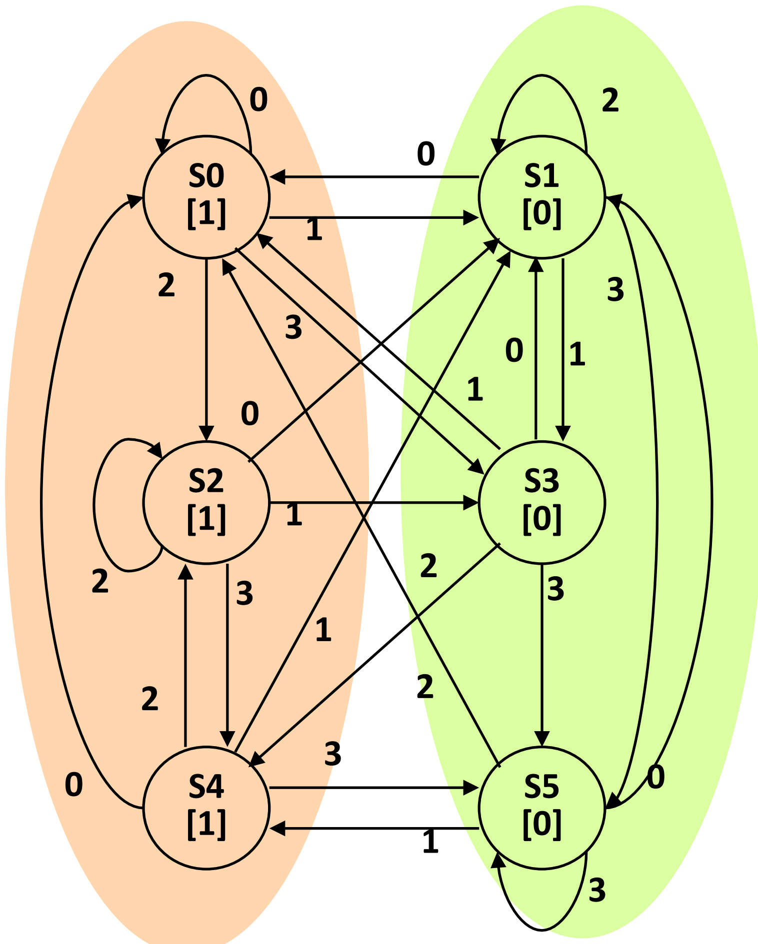
S2	S1	S3	S2	S4	1
S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

state
transition table

Put states into groups based on their outputs (or whether they accept or reject)

State Minimization Example



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S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

state
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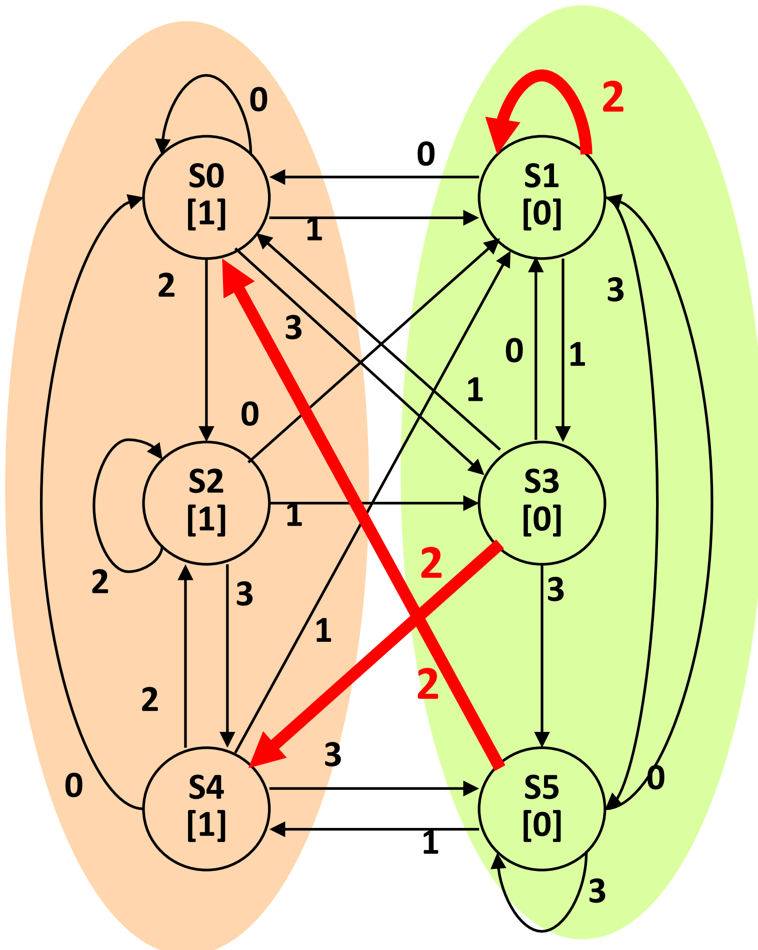
Put states into groups based on their outputs (or whether they accept or reject)

If there is a symbol **s** so that not all states in a group **G** agree on which group **s** leads to, split **G** based on which group the states go to on **s**

State Minimization Example



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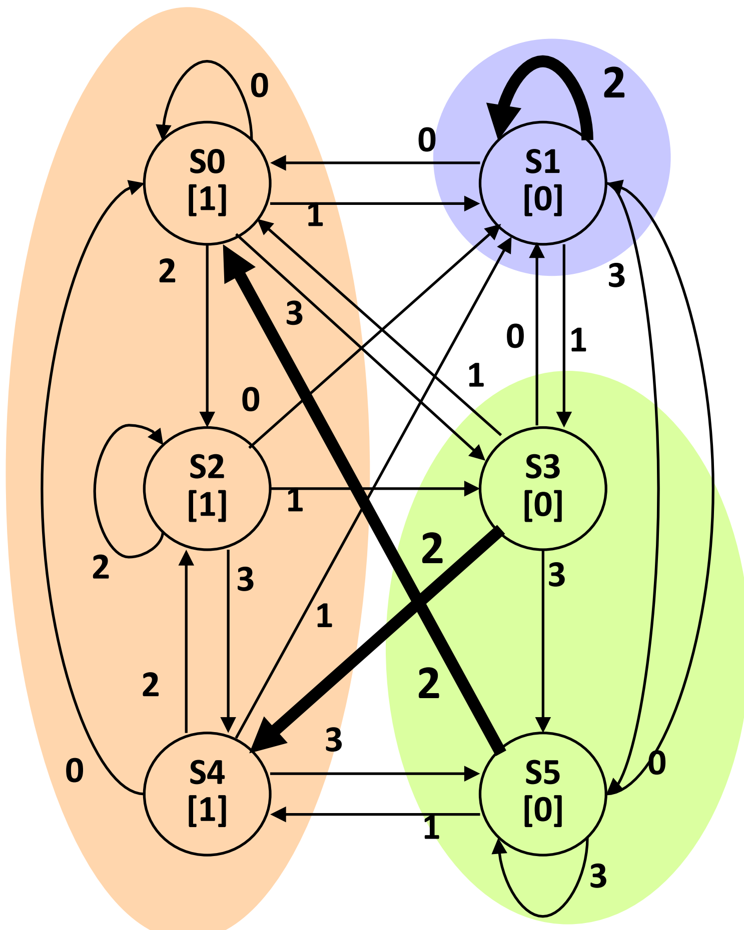

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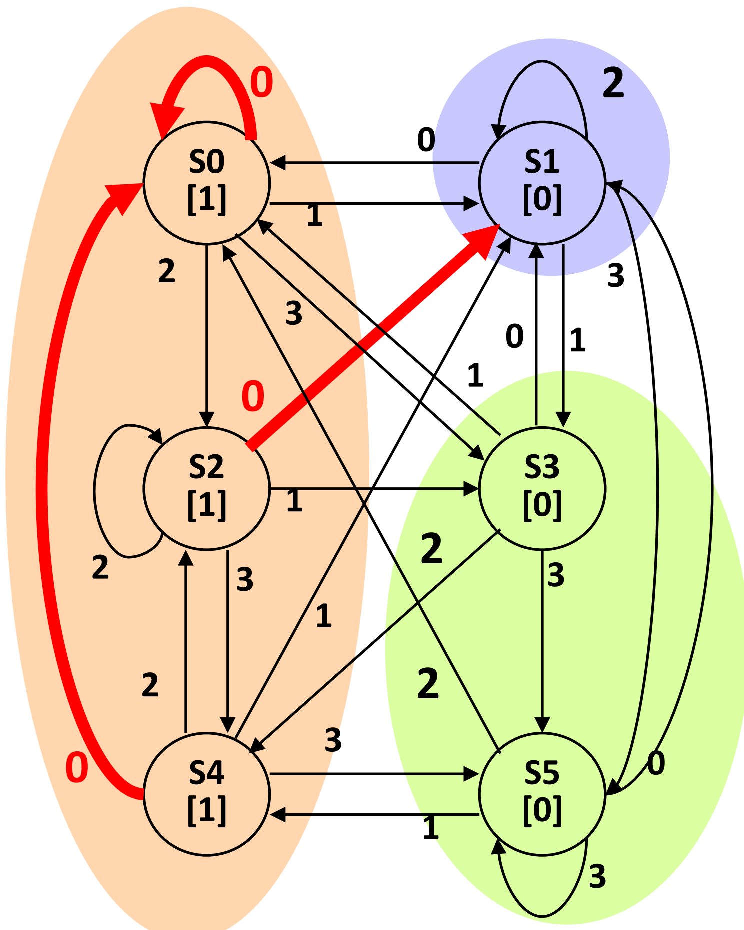

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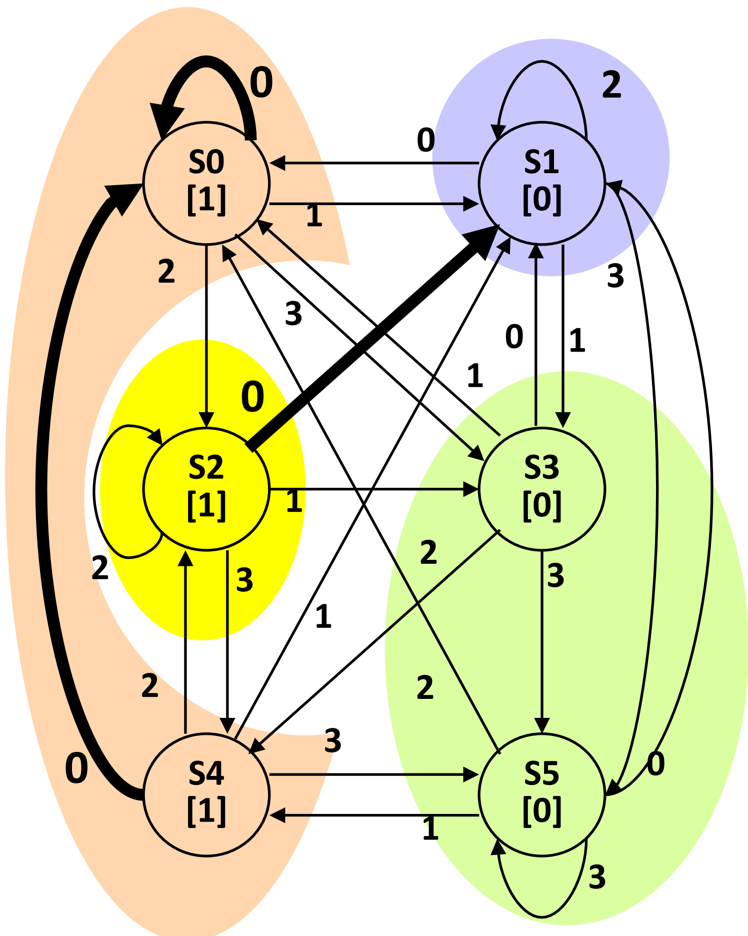

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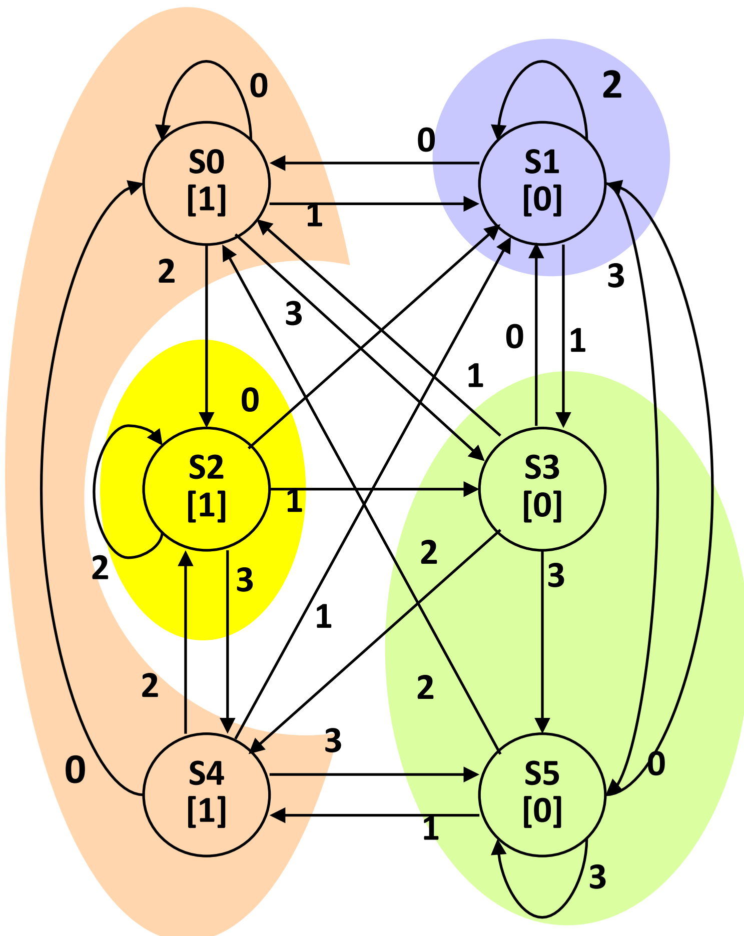

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S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

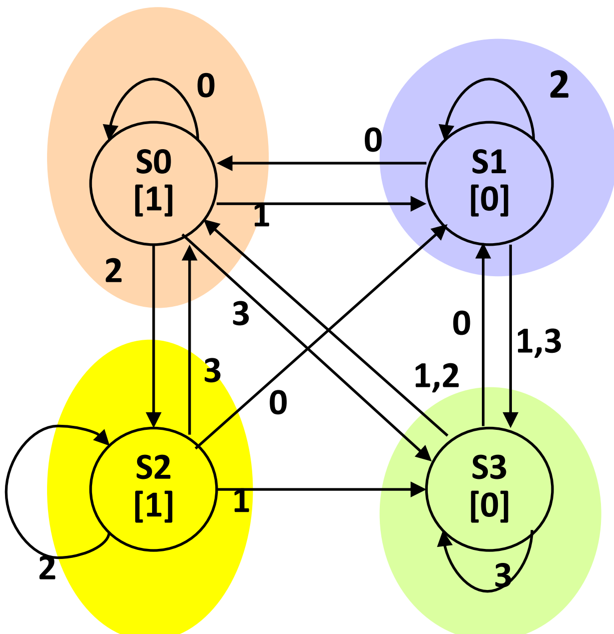
state
transition table

Finally convert groups to states:

Can combine states S0-S4 and S3-S5.

In table replace all S4 with S0 and all S5 with S3

Minimized Machine



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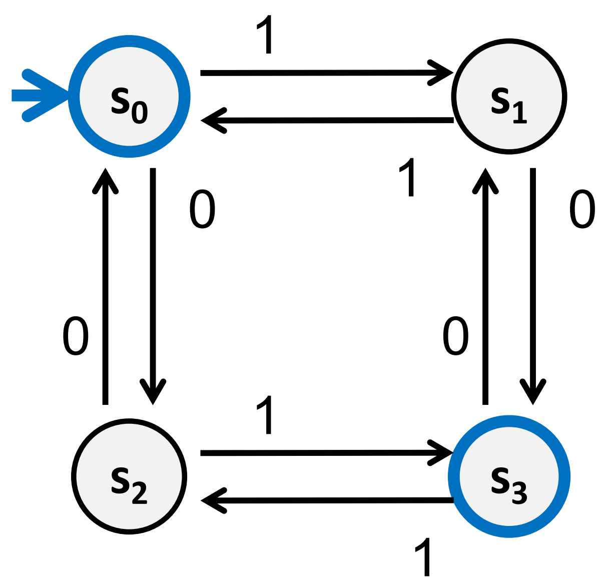
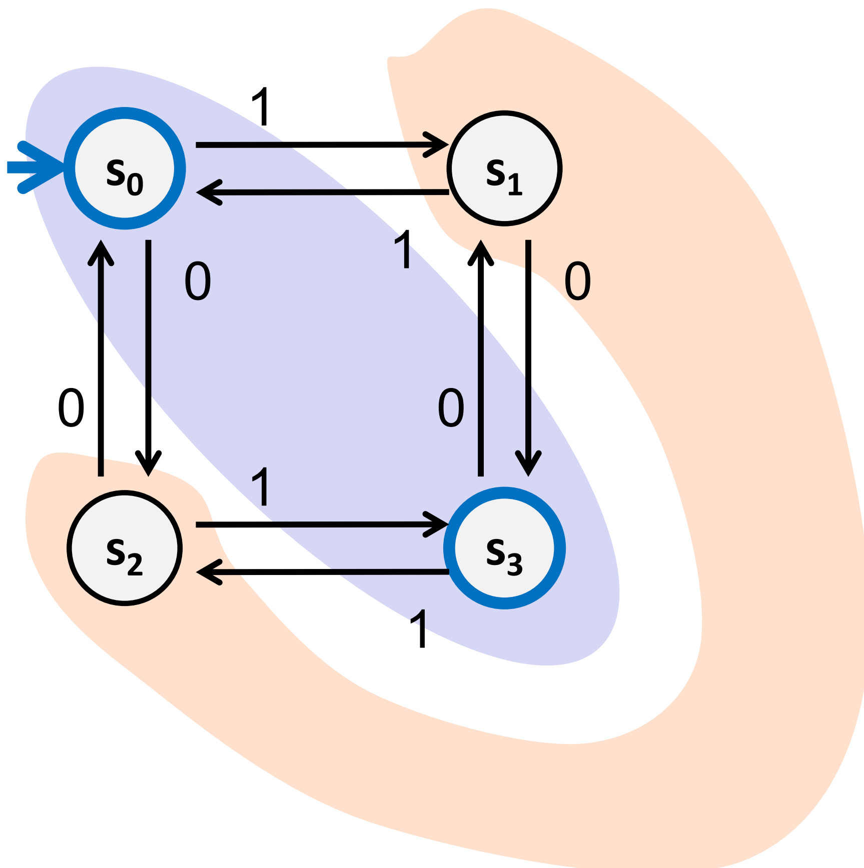
state
transition table

A Simpler Minimization Example



The set of all binary strings with # of 1's ≡ # of 0's (mod 2).

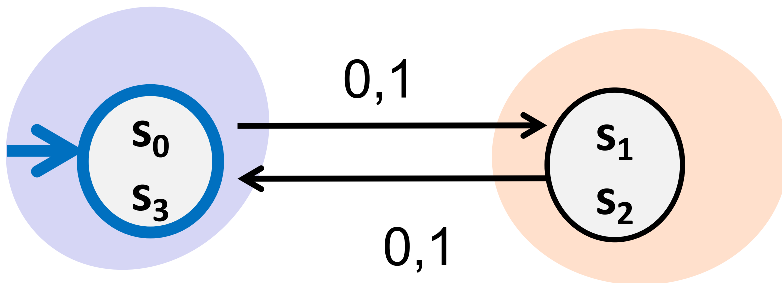
A Simpler Minimization Example



Split states into accept/reject groups

Every symbol causes the DFA to go from one group to the other so neither group needs to be split

Minimized DFA

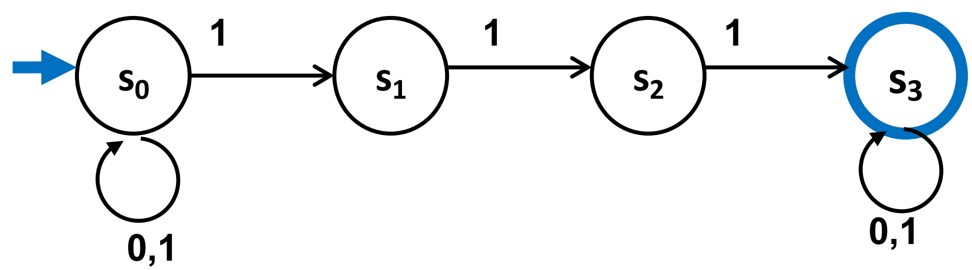


The set of all binary strings with # of 1's ≡ # of 0's (mod 2).

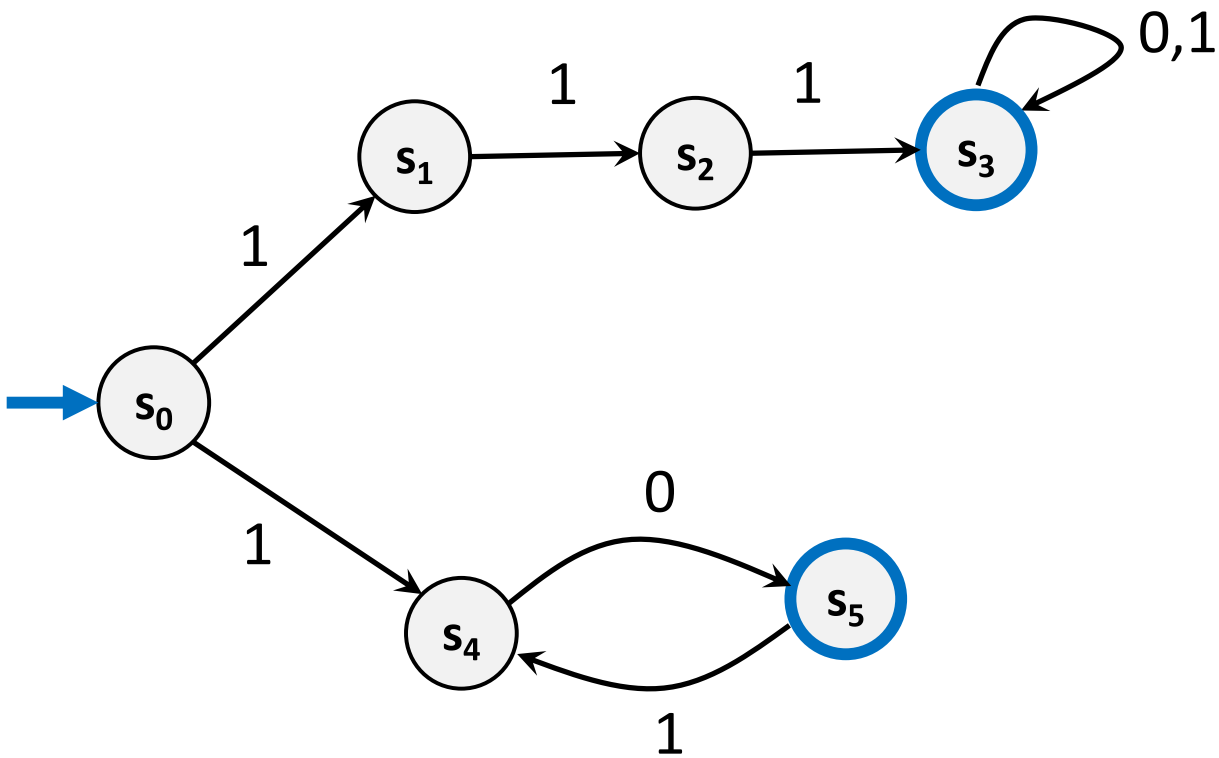
= The set of all binary strings with even length.

Nondeterministic Finite Automata (NFA)

- Graph with start state, final states, edges labeled by symbols (like DFA) but
  - Not required to have exactly 1 edge out of each state labeled by each symbol— can have 0 or >1
  - Also can have edges labeled by empty string ε
- **Definition:** x is in the language recognized by an NFA if and only if <u>some</u> valid execution of the machine gets to an accept state

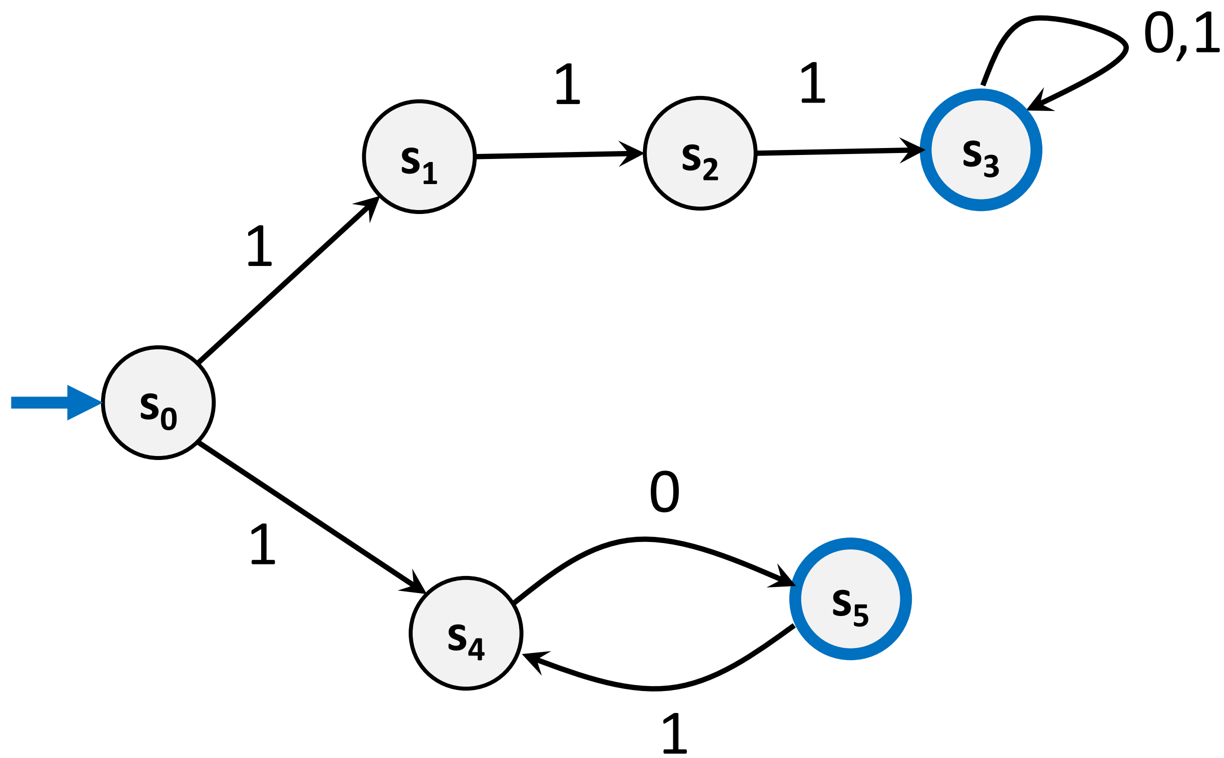


Consider This NFA



What language does this NFA accept?

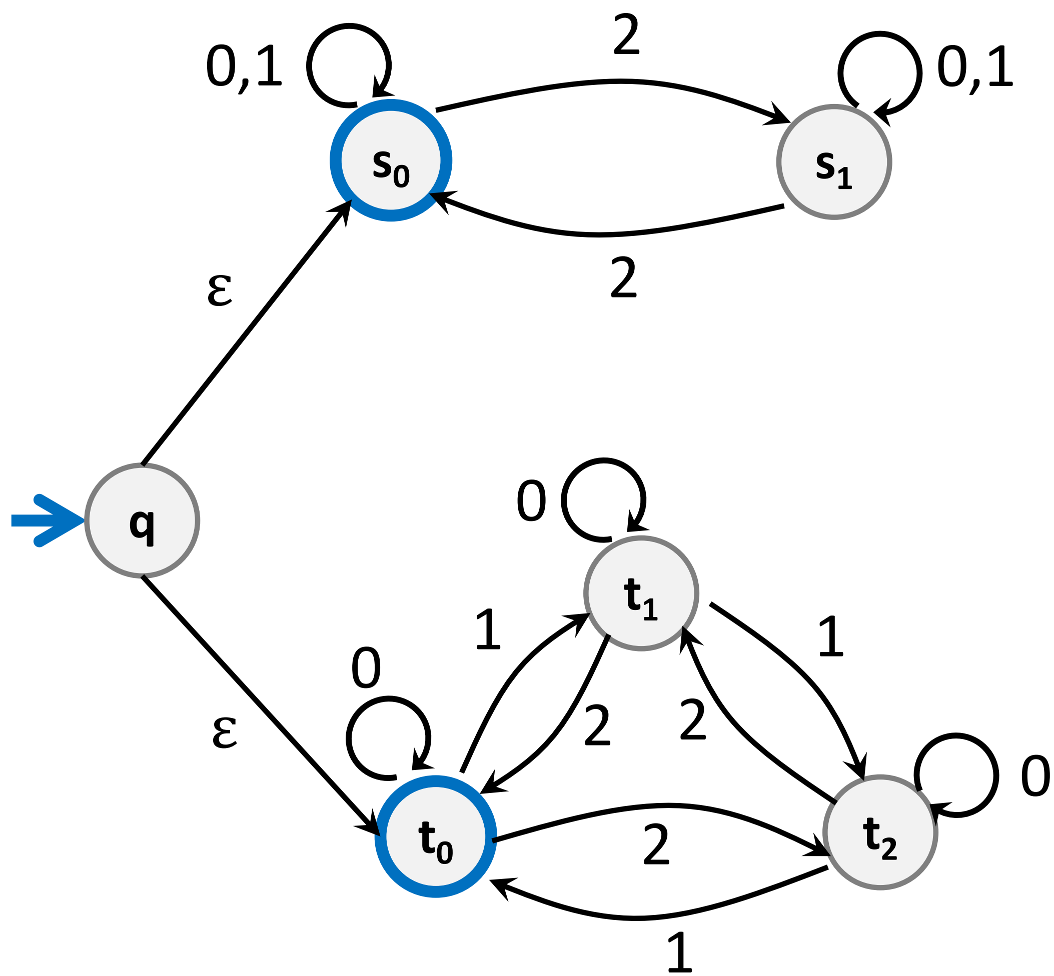
Consider This NFA



What language does this NFA accept?

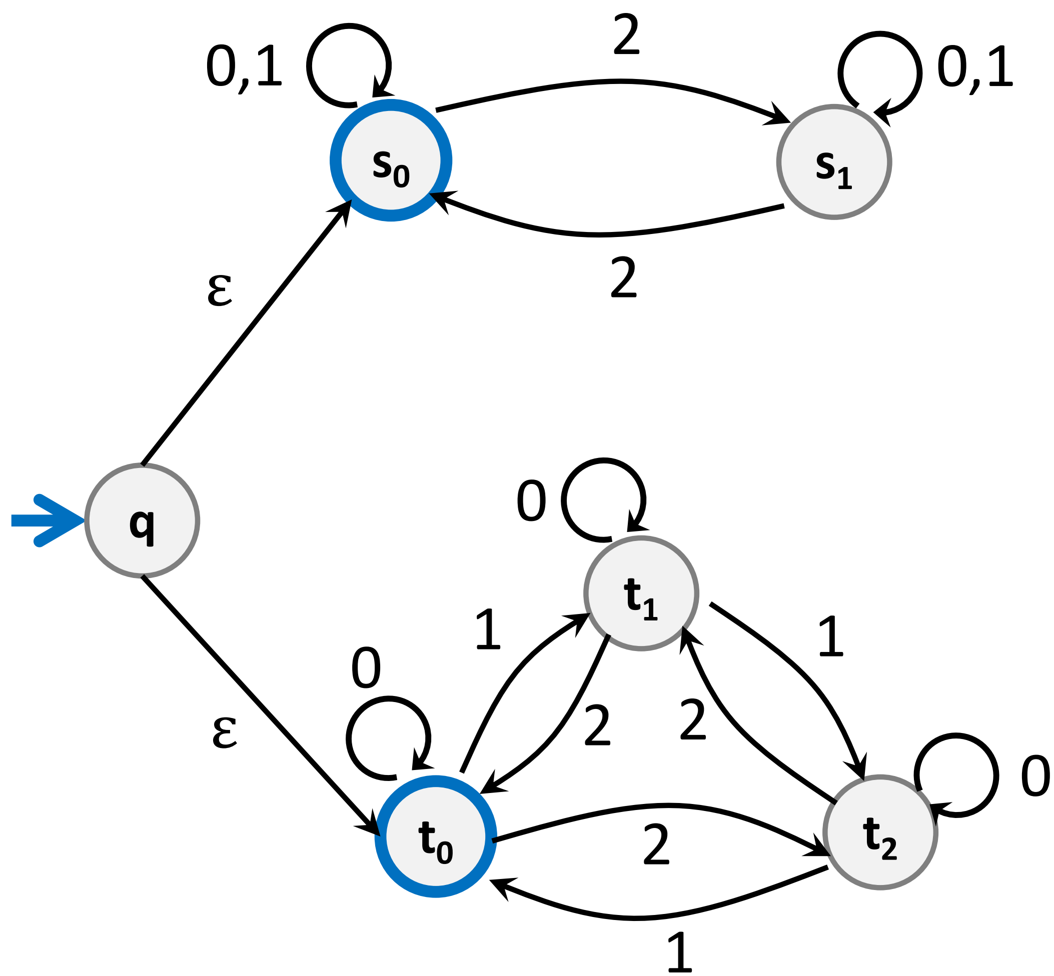
10(10)* U 111 (0 U 1)*

NFA ε-moves



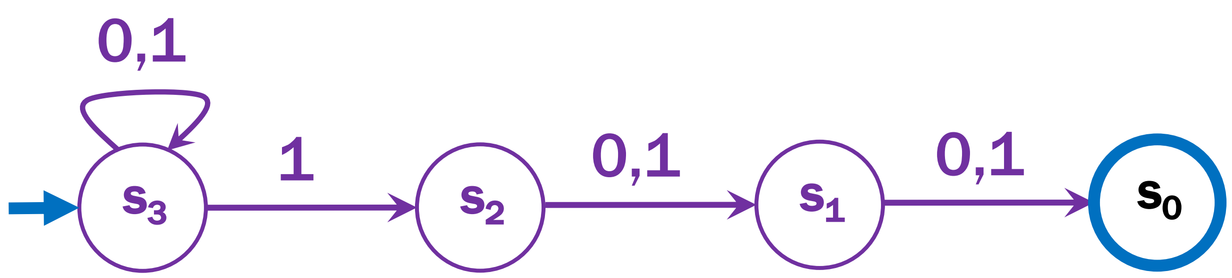
NFA ε-moves

Strings over {0,1,2} w/even # of 2's OR sum to 0 mod 3

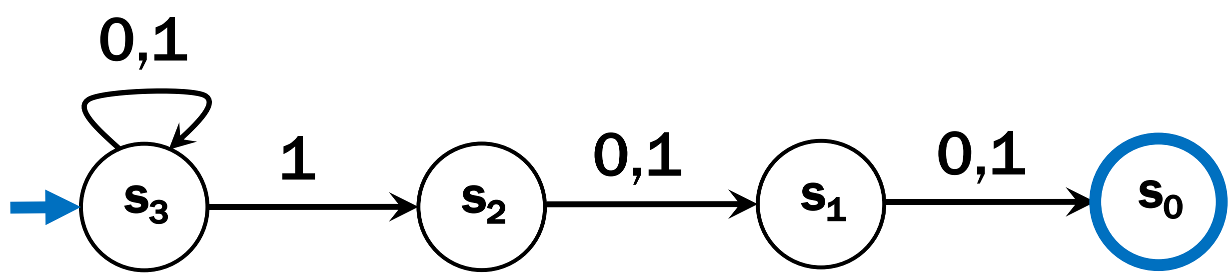


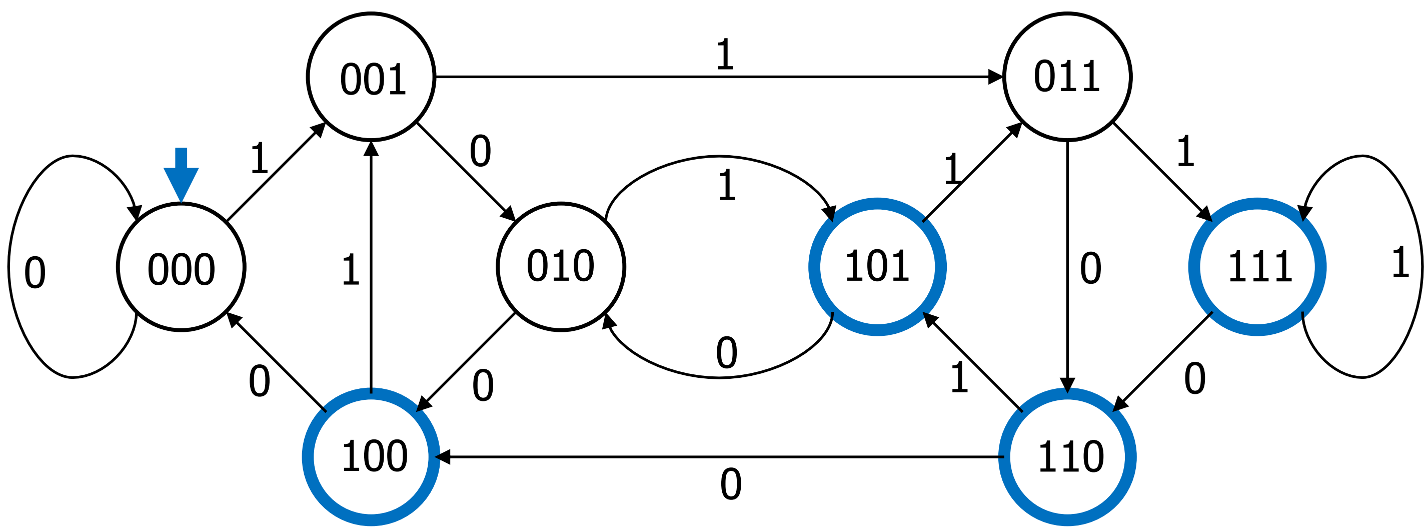
NFA for set of binary strings with a 1 in the 3rd position from the end

NFA for set of binary strings with a 1 in the 3rd position from the end



Compare with the smallest DFA





Summary of NFAs

- Generalization of DFAs
  - drop two restrictions of DFAs
  - every DFA <u>is</u> an NFA

- *Seem* to be more powerful
  - designing is easier than with DFAs

- *Seem* related to regular expressions