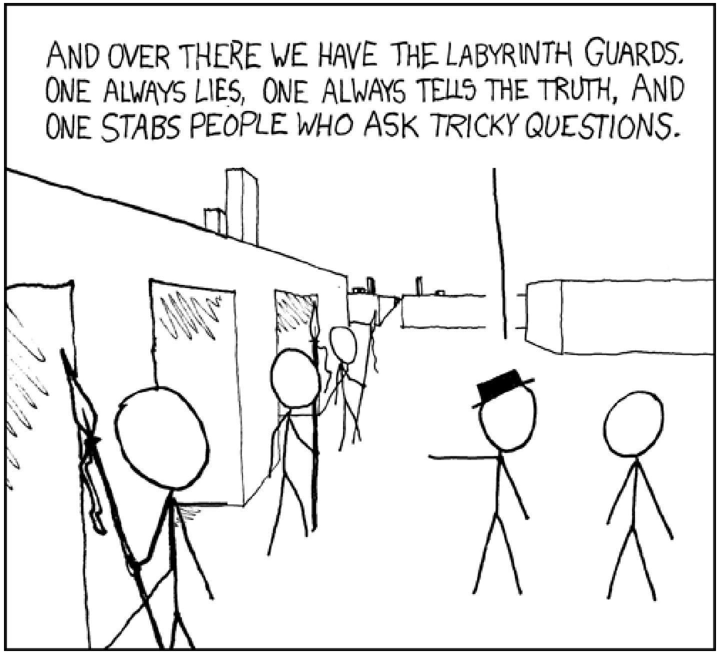
CSE 311: Foundations of Computing

Lecture 3: Digital Circuits & Equivalence



Recap from last class

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \to r \equiv \neg q \vee r$

Proving equivalence

- **Identity**
  - $q \wedge T \equiv q$
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- **Domination**
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  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
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  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

Proving equivalence

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
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  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
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- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

Example: Show that $\neg p \vee (p \vee p) \equiv T$

$$\neg p \vee (p \vee p) \equiv (\qquad\qquad\qquad\qquad)$$
$$\equiv (\qquad\qquad\qquad\qquad)$$
$$\equiv \mathbf{T}$$

Proving equivalence

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
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  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
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- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

Example: Show that $\neg p \vee (p \vee p) \equiv T$

$\neg p \vee (p \vee p) \equiv (\quad \neg p \vee p \quad)$ Idempotent

$\equiv (\qquad \qquad)$

$\equiv$ **T**

Proving equivalence

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
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  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
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- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

Example: Show that $\neg p \vee (p \vee p) \equiv T$

$\neg p \vee (p \vee p) \equiv (\quad \neg p \vee p \quad)$ Idempotent

$\equiv (\quad p \vee \neg p \quad)$ Commutative

$\equiv$ **T**

Proving equivalence

- **Identity**
  - $q \land T \equiv q$
  - $q \lor F \equiv q$
- **Domination**
  - $q \lor T \equiv T$
  - $q \land F \equiv F$
- **Idempotent**
  - $q \lor q \equiv q$
  - $q \land q \equiv q$
- **Commutative**
  - $q \lor r \equiv r \lor q$
  - $q \land r \equiv r \land q$
- **De Morgan Laws**
  - $\neg(q \land r) \equiv \neg q \lor \neg r$
  - $\neg(q \lor r) \equiv \neg q \land \neg r$
- **Associative**
  - $(q \lor r) \lor s \equiv q \lor (r \lor s)$
  - $(q \land r) \land s \equiv q \land (r \land s)$
- **Distributive**
  - $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$
  - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$
- **Absorption**
  - $q \lor (q \land r) \equiv q$
  - $q \land (q \lor r) \equiv q$
- **Negation**
  - $q \lor \neg q \equiv T$
  - $q \land \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \lor r$

One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

Example: Show that $\neg p \lor (p \lor p) \equiv T$

$\neg p \lor (p \lor p) \equiv (\quad \neg p \lor p \quad)$ Idempotent

$\equiv (\quad p \lor \neg p \quad)$ Commutative

$\equiv$ **T** Negation

What is a proof?

A proof is a logical argument that guarantees the conclusion is true. In this case, the conclusion is

$$\neg p \vee (p \vee p) \equiv \mathsf{T}$$

$\neg p \vee (p \vee p)$	$\equiv (\quad \neg p \vee p \quad)$	Idempotent
	$\equiv (\quad p \vee \neg p \quad)$	Commutative
	$\equiv \mathbf{T}$	Negation

What is a proof?

A proof is a logical argument that guarantees the conclusion is true. In this case, the conclusion is

$$\neg p \vee (p \vee p) \equiv \mathsf{T}$$

The syntax there is a little terse. In full, it means:

(1) $\neg p \vee (p \vee p) \equiv \neg p \vee p$ by the Idempotent rule,

(2) $\neg p \vee p \equiv p \vee \neg p$ by the Commutative rule, and

(3) $p \vee \neg p \equiv \mathsf{T}$ by the Negation rule.

Therefore, we conclude $\neg p \vee (p \vee p) \equiv \mathsf{T}$

$$\begin{aligned} \neg p \vee (p \vee p) &\equiv (\quad \neg p \vee p \quad) && \text{Idempotent} \\ &\equiv (\quad p \vee \neg p \quad) && \text{Commutative} \\ &\equiv \mathbf{T} && \text{Negation} \end{aligned}$$

Analyzing the Garfield Sentence with a Truth Table

Why not just use a truth table?

q	r	s	$\neg s$	$r \vee \neg s$	$r \wedge s$	$(r \wedge s) \rightarrow q$	$((r \wedge s) \rightarrow q) \wedge (r \vee \neg s)$
F	F	F	T	T	F	T	T
F	F	T	F	F	F	T	F
F	T	F	T	T	F	T	T
F	T	T	F	T	T	F	F
T	F	F	T	T	F	T	T
T	F	T	F	F	F	T	F
T	T	F	T	T	F	T	T
T	T	T	F	T	T	T	T

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

q	***r***	***q→r***
T	T	T
T	F	F
F	T	T
F	F	T

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

q	r	q→r
T	T	T
T	F	F
F	T	T
F	F	T

$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv \neg q \vee r$

- **Identity**
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- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
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  - $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$
$$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)] \quad \textbf{Associative}$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv \neg q \vee r$$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

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- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$

$$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$$ **Associative**

$$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$$ **Distributive**

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$$\equiv \neg q \vee r$$

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  - $q \vee r \equiv r \vee q$
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  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
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  - $q \vee (q \wedge r) \equiv q$
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- **Negation**
  - $q \vee \neg q \equiv T$
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- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$

$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$ **Associative**

$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$ **Distributive**

$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}]$ **Negation**

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv \neg q \vee r$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

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  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
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  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
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- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
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- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$

$$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)] \quad \textbf{Associative}$$

$$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)] \quad \textbf{Distributive}$$

$$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}] \quad \textbf{Negation}$$

$$\equiv (q \wedge r) \vee [\neg q] \quad \textbf{Identity}$$

$$\equiv$$

$$\equiv$$

$$\equiv$$

$$\equiv$$

$$\equiv$$

$$\equiv \neg q \vee r$$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

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  - $q \vee r \equiv r \vee q$
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- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$

$$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$$ **Associative**

$$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$$ **Distributive**

$$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}]$$ **Negation**

$$\equiv (q \wedge r) \vee [\neg q]$$ **Identity**

$$\equiv [\neg q] \vee (q \wedge r)$$ **Commutative**

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$$\equiv \neg q \vee r$$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

q no longer matters in $q \wedge r$ if $\neg q$ automatically makes the expression true.

- **Identity**
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  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
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- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$	$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$	**Associative**
	$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$	**Distributive**
	$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}]$	**Negation**
	$\equiv (q \wedge r) \vee [\neg q]$	**Identity**
q no longer matters in $q \wedge r$ if $\neg q$ automatically makes the expression true.	$\equiv [\neg q] \vee (q \wedge r)$	**Commutative**
	$\equiv (\neg q \vee q) \wedge (\neg q \vee r)$	**Distributive**
	$\equiv$	
	$\equiv$	
	$\equiv$	
	$\equiv \neg q \vee r$	

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$

$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$ **Associative**

$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$ **Distributive**

$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}]$ **Negation**

$\equiv (q \wedge r) \vee [\neg q]$ **Identity**

$\equiv [\neg q] \vee (q \wedge r)$ **Commutative**

$\equiv (\neg q \vee q) \wedge (\neg q \vee r)$ **Distributive**

$\equiv (q \vee \neg q) \wedge (\neg q \vee r)$ **Commutative**

$\equiv$

$\equiv$

$\equiv \neg q \vee r$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

q no longer matters in $q \wedge r$ if $\neg q$ automatically makes the expression true.

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$

$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$ **Associative**

$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$ **Distributive**

$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}]$ **Negation**

$\equiv (q \wedge r) \vee [\neg q]$ **Identity**

$\equiv [\neg q] \vee (q \wedge r)$ **Commutative**

$\equiv (\neg q \vee q) \wedge (\neg q \vee r)$ **Distributive**

$\equiv (q \vee \neg q) \wedge (\neg q \vee r)$ **Commutative**

$\equiv \mathrm{T} \wedge (\neg q \vee r)$ **Negation**

$\equiv$

$\equiv \neg q \vee r$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

q no longer matters in $q \wedge r$ if $\neg q$ automatically makes the expression true.

- **Identity**
- $q \wedge T \equiv q$
- $q \vee F \equiv q$
- **Domination**
- $q \vee T \equiv T$
- $q \wedge F \equiv F$
- **Idempotent**
- $q \vee q \equiv q$
- $q \wedge q \equiv q$
- **Commutative**
- $q \vee r \equiv r \vee q$
- $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
- $\neg(q \wedge r) \equiv \neg q \vee \neg r$
- $\neg(q \vee r) \equiv \neg q \wedge \neg r$

- **Associative**
- $(q \vee r) \vee s \equiv q \vee (r \vee s)$
- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
- $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
- $q \vee (q \wedge r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- **Negation**
- $q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$

$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$	**Associative**
$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$	**Distributive**
$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}]$	**Negation**
$\equiv (q \wedge r) \vee [\neg q]$	**Identity**
$\equiv [\neg q] \vee (q \wedge r)$	**Commutative**
$\equiv (\neg q \vee q) \wedge (\neg q \vee r)$	**Distributive**
$\equiv (q \vee \neg q) \wedge (\neg q \vee r)$	**Commutative**
$\equiv \mathrm{T} \wedge (\neg q \vee r)$	**Negation**
$\equiv (\neg q \vee r) \wedge \mathrm{T}$	**Commutative**
$\equiv \neg q \vee r$	

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

q no longer matters in $q \wedge r$ if $\neg q$ automatically makes the expression true.

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$

$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r)$	
$\equiv (q \land r) \lor [(\neg q \land r) \lor (\neg q \land \neg r)]$	Associative
$\equiv (q \land r) \lor [\neg q \land (r \lor \neg r)]$	Distributive
$\equiv (q \land r) \lor [\neg q \land \mathrm{T}]$	Negation
$\equiv (q \land r) \lor [\neg q]$	Identity
$\equiv [\neg q] \lor (q \land r)$	Commutative
$\equiv (\neg q \lor q) \land (\neg q \lor r)$	Distributive
$\equiv (q \lor \neg q) \land (\neg q \lor r)$	Commutative
$\equiv \mathrm{T} \land (\neg q \lor r)$	Negation
$\equiv (\neg q \lor r) \land \mathrm{T}$	Commutative
$\equiv \neg q \lor r$	Identity

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

q no longer matters in $q \land r$ if $\neg q$ automatically makes the expression true.

- **Identity**
  - $q \land T \equiv q$
  - $q \lor F \equiv q$
- **Domination**
  - $q \lor T \equiv T$
  - $q \land F \equiv F$
- **Idempotent**
  - $q \lor q \equiv q$
  - $q \land q \equiv q$
- **Commutative**
  - $q \lor r \equiv r \lor q$
  - $q \land r \equiv r \land q$
- **De Morgan Laws**
  - $\neg(q \land r) \equiv \neg q \lor \neg r$
  - $\neg(q \lor r) \equiv \neg q \land \neg r$
- **Associative**
  - $(q \lor r) \lor s \equiv q \lor (r \lor s)$
  - $(q \land r) \land s \equiv q \land (r \land s)$
- **Distributive**
  - $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$
  - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$
- **Absorption**
  - $q \lor (q \land r) \equiv q$
  - $q \land (q \lor r) \equiv q$
- **Negation**
  - $q \lor \neg q \equiv T$
  - $q \land \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \to r \equiv \neg q \lor r$

Prove this is a Tautology

- **Associative**
- $(q \vee r) \vee s \equiv q \vee (r \vee s)$
- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
- $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
- $q \vee (q \wedge r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- **Negation**
- $q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \rightarrow r \equiv \neg q \vee r$

$$(q \wedge r) \rightarrow (r \vee q)$$

- **Identity**
- $q \wedge T \equiv q$
- $q \vee F \equiv q$
- **Domination**
- $q \vee T \equiv T$
- $q \wedge F \equiv F$
- **Idempotent**
- $q \vee q \equiv q$
- $q \wedge q \equiv q$
- **Commutative**
- $q \vee r \equiv r \vee q$
- $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
- $\neg(q \wedge r) \equiv \neg q \vee \neg r$
- $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Prove this is a Tautology

- **Associative**
- $(q \vee r) \vee s \equiv q \vee (r \vee s)$
- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
- $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
- $q \vee (q \wedge r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- **Negation**
- $q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \rightarrow r \equiv \neg q \vee r$

$$(q \wedge r) \rightarrow (r \vee q)$$

Use a series of equivalences:

$$(q \wedge r) \rightarrow (r \vee q) \equiv$$

$$\equiv$$

$$\equiv$$

$$\equiv$$

$$\equiv$$

$$\equiv$$

$$\equiv$$

$$\equiv$$

$$\equiv \quad \mathbf{T}$$

- **Identity**
- $q \wedge T \equiv q$
- $q \vee F \equiv q$
- **Domination**
- $q \vee T \equiv T$
- $q \wedge F \equiv F$
- **Idempotent**
- $q \vee q \equiv q$
- $q \wedge q \equiv q$
- **Commutative**
- $q \vee r \equiv r \vee q$
- $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
- $\neg(q \wedge r) \equiv \neg q \vee \neg r$
- $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace $\rightarrow$; move $\neg$ inside; simplify

Prove this is a Tautology

- **Associative**
- $(q \vee r) \vee s \equiv q \vee (r \vee s)$
- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
- $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
- $q \vee (q \wedge r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- **Negation**
- $q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \to r \equiv \neg q \vee r$

$$(q \wedge r) \to (r \vee q)$$

Use a series of equivalences:

$$(q \wedge r) \to (r \vee q) \equiv \neg(q \wedge r) \vee (r \vee q)$$ Law of Implication

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$ **T**

- **Identity**
- $q \wedge T \equiv q$
- $q \vee F \equiv q$
- **Domination**
- $q \vee T \equiv T$
- $q \wedge F \equiv F$
- **Idempotent**
- $q \vee q \equiv q$
- $q \wedge q \equiv q$
- **Commutative**
- $q \vee r \equiv r \vee q$
- $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
- $\neg(q \wedge r) \equiv \neg q \vee \neg r$
- $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace $\to$; move $\neg$ inside; simplify

Prove this is a Tautology

- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \to r \equiv \neg q \vee r$

$$(q \wedge r) \to (r \vee q)$$

Use a series of equivalences:

$(q \wedge r) \to (r \vee q) \equiv \neg(q \wedge r) \vee (r \vee q)$ — **Law of Implication**

$\equiv (\neg q \vee \neg r) \vee (r \vee q)$ — **DeMorgan**

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$ **T**

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace $\to$; move $\neg$ inside; simplify

Prove this is a Tautology

- **Associative**
- $(q \lor r) \lor s \equiv q \lor (r \lor s)$
- $(q \land r) \land s \equiv q \land (r \land s)$
- **Distributive**
- $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$
- $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$
- **Absorption**
- $q \lor (q \land r) \equiv q$
- $q \land (q \lor r) \equiv q$
- **Negation**
- $q \lor \neg q \equiv T$
- $q \land \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \to r \equiv \neg q \lor r$

$$(q \land r) \to (r \lor q)$$

Use a series of equivalences:

$$(q \land r) \to (r \lor q) \equiv \neg(q \land r) \lor (r \lor q)$$ Law of Implication

$$\equiv (\neg q \lor \neg r) \lor (r \lor q)$$ DeMorgan

$$\equiv \neg q \lor (\neg r \lor (r \lor q))$$ Associative

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$ **T**

- **Identity**
- $q \land T \equiv q$
- $q \lor F \equiv q$
- **Domination**
- $q \lor T \equiv T$
- $q \land F \equiv F$
- **Idempotent**
- $q \lor q \equiv q$
- $q \land q \equiv q$
- **Commutative**
- $q \lor r \equiv r \lor q$
- $q \land r \equiv r \land q$
- **De Morgan Laws**
- $\neg(q \land r) \equiv \neg q \lor \neg r$
- $\neg(q \lor r) \equiv \neg q \land \neg r$

Our strategy: Replace $\to$; move $\neg$ inside; simplify

Prove this is a Tautology

- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \to r \equiv \neg q \vee r$

$$(q \wedge r) \to (r \vee q)$$

Use a series of equivalences:

$(q \wedge r) \to (r \vee q) \equiv \neg(q \wedge r) \vee (r \vee q)$	Law of Implication
$\equiv (\neg q \vee \neg r) \vee (r \vee q)$	DeMorgan
$\equiv \neg q \vee (\neg r \vee (r \vee q))$	Associative
$\equiv \neg q \vee ((\neg r \vee r) \vee q)$	Associative
$\equiv$	
$\equiv$	
$\equiv$	
$\equiv$	
$\equiv$ **T**	

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace $\to$; move $\neg$ inside; simplify

Prove this is a Tautology

- **Associative**
- $(q \vee r) \vee s \equiv q \vee (r \vee s)$
- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
- $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
- $q \vee (q \wedge r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- **Negation**
- $q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \to r \equiv \neg q \vee r$

$$(q \wedge r) \to (r \vee q)$$

Use a series of equivalences:

$$
\begin{aligned}
(q \wedge r) \to (r \vee q) &\equiv \neg(q \wedge r) \vee (r \vee q) && \text{Law of Implication} \\
&\equiv (\neg q \vee \neg r) \vee (r \vee q) && \text{DeMorgan} \\
&\equiv \neg q \vee (\neg r \vee (r \vee q)) && \text{Associative} \\
&\equiv \neg q \vee ((\neg r \vee r) \vee q) && \text{Associative} \\
&\equiv \neg q \vee (q \vee (\neg r \vee r)) && \text{Commutative} \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \ \mathbf{T}
\end{aligned}
$$

- **Identity**
- $q \wedge T \equiv q$
- $q \vee F \equiv q$
- **Domination**
- $q \vee T \equiv T$
- $q \wedge F \equiv F$
- **Idempotent**
- $q \vee q \equiv q$
- $q \wedge q \equiv q$
- **Commutative**
- $q \vee r \equiv r \vee q$
- $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
- $\neg(q \wedge r) \equiv \neg q \vee \neg r$
- $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace $\to$; move $\neg$ inside; simplify

Prove this is a Tautology

- Associative
- $(q \vee r) \vee s \equiv q \vee (r \vee s)$
- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- Distributive
- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
- $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- Absorption
- $q \vee (q \wedge r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- Negation
- $q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- Double negation
- $\neg(\neg q) \equiv q$
- Law of implication
- $q \rightarrow r \equiv \neg q \vee r$

$$(q \wedge r) \rightarrow (r \vee q)$$

Use a series of equivalences:

$(q \wedge r) \rightarrow (r \vee q) \equiv \neg(q \wedge r) \vee (r \vee q)$ Law of Implication

$\equiv (\neg q \vee \neg r) \vee (r \vee q)$ DeMorgan

$\equiv \neg q \vee (\neg r \vee (r \vee q))$ Associative

$\equiv \neg q \vee ((\neg r \vee r) \vee q)$ Associative

$\equiv \neg q \vee (q \vee (\neg r \vee r))$ Commutative

$\equiv (\neg q \vee q) \vee (\neg r \vee r)$ Associative

$\equiv$

$\equiv$

$\equiv$ **T**

- Identity
- $q \wedge T \equiv q$
- $q \vee F \equiv q$
- Domination
- $q \vee T \equiv T$
- $q \wedge F \equiv F$
- Idempotent
- $q \vee q \equiv q$
- $q \wedge q \equiv q$
- Commutative
- $q \vee r \equiv r \vee q$
- $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg(q \wedge r) \equiv \neg q \vee \neg r$
- $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace $\rightarrow$; move $\neg$ inside; simplify

Prove this is a Tautology

- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \to r \equiv \neg q \vee r$

$$(q \wedge r) \to (r \vee q)$$

Use a series of equivalences:

$(q \wedge r) \to (r \vee q) \equiv \neg(q \wedge r) \vee (r \vee q)$	Law of Implication
$\equiv (\neg q \vee \neg r) \vee (r \vee q)$	DeMorgan
$\equiv \neg q \vee (\neg r \vee (r \vee q))$	Associative
$\equiv \neg q \vee ((\neg r \vee r) \vee q)$	Associative
$\equiv \neg q \vee (q \vee (\neg r \vee r))$	Commutative
$\equiv (\neg q \vee q) \vee (\neg r \vee r)$	Associative
$\equiv (q \vee \neg q) \vee (r \vee \neg r)$	Commutative (twice)
$\equiv$	
$\equiv$ **T**	

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace $\to$; move $\neg$ inside; simplify

Prove this is a Tautology

- **Associative**
- $(q \vee r) \vee s \equiv q \vee (r \vee s)$
- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
- $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
- $q \vee (q \wedge r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- **Negation**
- $q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \rightarrow r \equiv \neg q \vee r$

$$(q \wedge r) \rightarrow (r \vee q)$$

Use a series of equivalences:

$(q \wedge r) \rightarrow (r \vee q) \equiv \neg(q \wedge r) \vee (r \vee q)$	Law of Implication
$\equiv (\neg q \vee \neg r) \vee (r \vee q)$	DeMorgan
$\equiv \neg q \vee (\neg r \vee (r \vee q))$	Associative
$\equiv \neg q \vee ((\neg r \vee r) \vee q)$	Associative
$\equiv \neg q \vee (q \vee (\neg r \vee r))$	Commutative
$\equiv (\neg q \vee q) \vee (\neg r \vee r)$	Associative
$\equiv (q \vee \neg q) \vee (r \vee \neg r)$	Commutative (twice)
$\equiv \mathbf{T} \vee \mathbf{T}$	Negation (twice)
$\equiv \mathbf{T}$	

- **Identity**
- $q \wedge T \equiv q$
- $q \vee F \equiv q$
- **Domination**
- $q \vee T \equiv T$
- $q \wedge F \equiv F$
- **Idempotent**
- $q \vee q \equiv q$
- $q \wedge q \equiv q$
- **Commutative**
- $q \vee r \equiv r \vee q$
- $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
- $\neg(q \wedge r) \equiv \neg q \vee \neg r$
- $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace $\rightarrow$; move $\neg$ inside; simplify

Prove this is a Tautology

- **Associative**
- $(q \lor r) \lor s \equiv q \lor (r \lor s)$
- $(q \land r) \land s \equiv q \land (r \land s)$
- **Distributive**
- $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$
- $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$
- **Absorption**
- $q \lor (q \land r) \equiv q$
- $q \land (q \lor r) \equiv q$
- **Negation**
- $q \lor \neg q \equiv T$
- $q \land \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \rightarrow r \equiv \neg q \lor r$

$$(q \land r) \rightarrow (r \lor q)$$

Use a series of equivalences:

$(q \land r) \rightarrow (r \lor q) \equiv \neg(q \land r) \lor (r \lor q)$	Law of Implication
$\equiv (\neg q \lor \neg r) \lor (r \lor q)$	DeMorgan
$\equiv \neg q \lor (\neg r \lor (r \lor q))$	Associative
$\equiv \neg q \lor ((\neg r \lor r) \lor q)$	Associative
$\equiv \neg q \lor (q \lor (\neg r \lor r))$	Commutative
$\equiv (\neg q \lor q) \lor (\neg r \lor r)$	Associative
$\equiv (q \lor \neg q) \lor (r \lor \neg r)$	Commutative (twice)
$\equiv \mathbf{T} \lor \mathbf{T}$	Negation (twice)
$\equiv \mathbf{T}$	Domination/Identity

- **Identity**
- $q \land T \equiv q$
- $q \lor F \equiv q$
- **Domination**
- $q \lor T \equiv T$
- $q \land F \equiv F$
- **Idempotent**
- $q \lor q \equiv q$
- $q \land q \equiv q$
- **Commutative**
- $q \lor r \equiv r \lor q$
- $q \land r \equiv r \land q$
- **De Morgan Laws**
- $\neg(q \land r) \equiv \neg q \lor \neg r$
- $\neg(q \lor r) \equiv \neg q \land \neg r$

Our strategy: Replace $\rightarrow$; move $\neg$ inside; simplify

Logical Proofs of Equivalence/Tautology

- Not smaller than truth tables when there are only a few propositional variables...

- ...but usually ***much shorter*** than truth table proofs when there are many propositional variables

- A big advantage will be that we can extend them to a more in-depth understanding of logic for which truth tables don't apply.

Lecture 3 Activity

- You will be assigned to **breakout rooms**. Please:
- Introduce yourself
- Choose someone to share screen, showing this PDF
- Show that $p \to q \equiv \neg q \to \neg p$ using a sequence of elementary equivalences.

Fill out a poll everywhere for Activity Credit!
Go to pollev.com/philipmg and login with your UW identity

$p \to q \equiv \neg p \vee q$ LoI
$\equiv q \vee \neg p$ Comm
$\equiv (\neg\neg q) \vee \neg p$ DN
$\equiv \neg q \to \neg p$ LoI

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \to r \equiv \neg q \vee r$

Digital Circuits

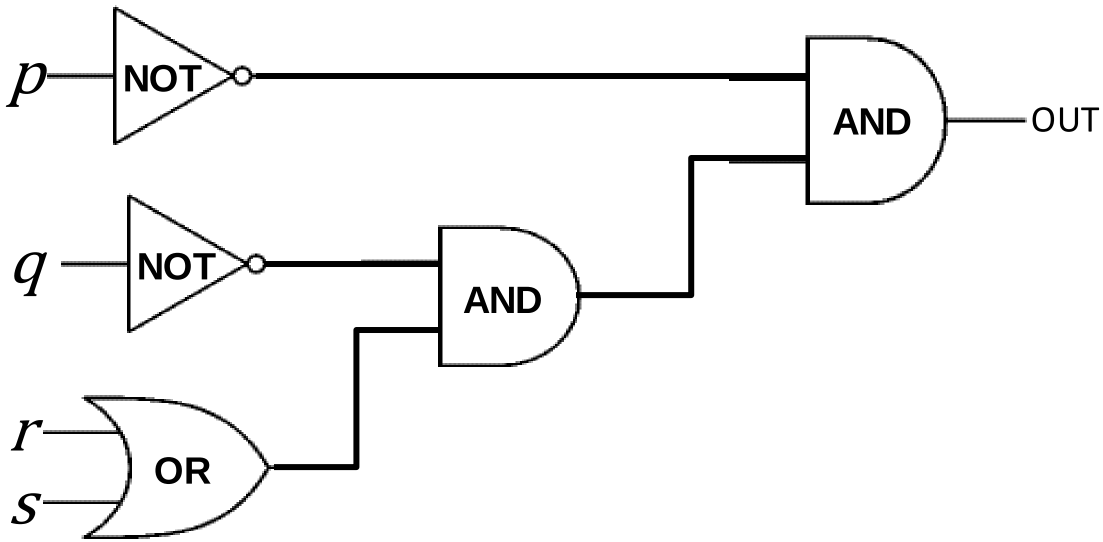
Computing With Logic

- T corresponds to 1 or "high" voltage
- F corresponds to 0 or "low" voltage

Gates

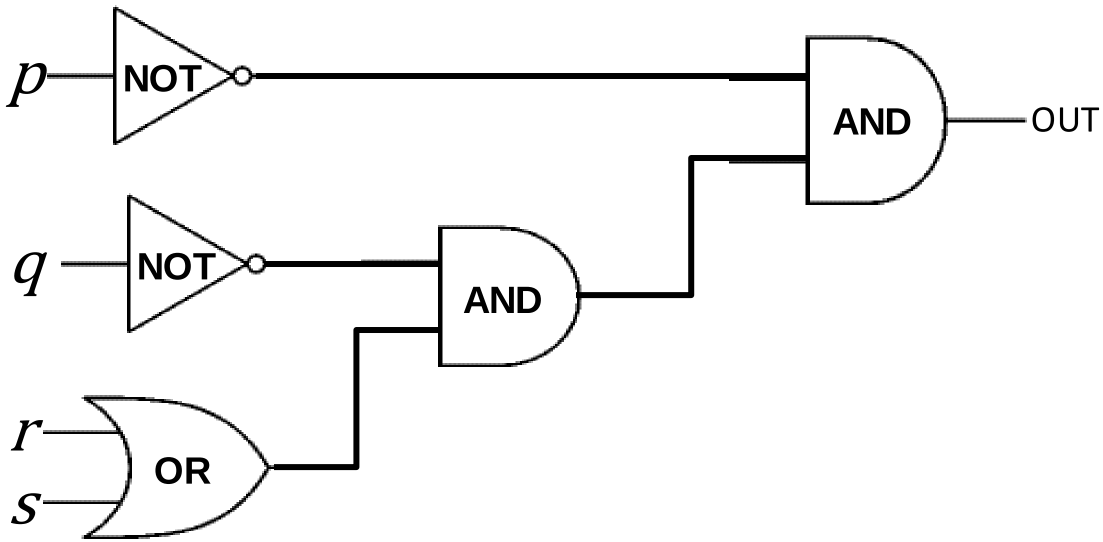
- Take inputs and produce outputs (functions)
- Several kinds of gates
- Correspond to propositional connectives (most of them)

Combinational Logic Circuits



Values get sent along wires connecting gates

Combinational Logic Circuits



Values get sent along wires connecting gates

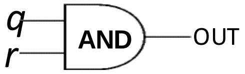
$$\neg p \wedge (\neg q \wedge (r \vee s))$$

And Gate

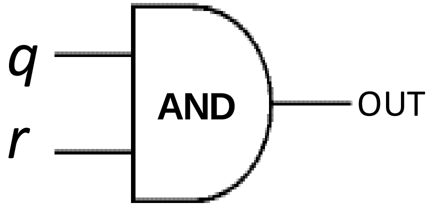
AND Connective vs. AND Gate

$q \wedge r$

q	r	$q \wedge r$
T	T	T
T	F	F
F	T	F
F	F	F



q	r	OUT
1	1	1
1	0	0
0	1	0
0	0	0



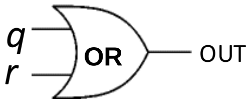
"block looks like D of AND"

Or Gate

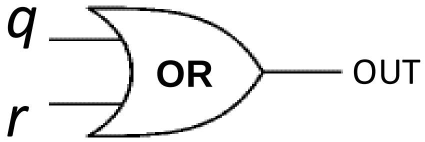
OR Connective vs. OR Gate

$q \vee r$

q	r	$q \vee r$
T	T	T
T	F	T
F	T	T
F	F	F



q	r	OUT
1	1	1
1	0	1
0	1	1
0	0	0

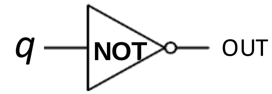


"arrowhead block looks like V"

Not Gates

NOT Connective vs. NOT Gate

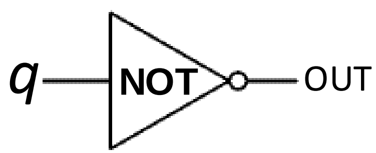
$\neg q$



Also called *inverter*

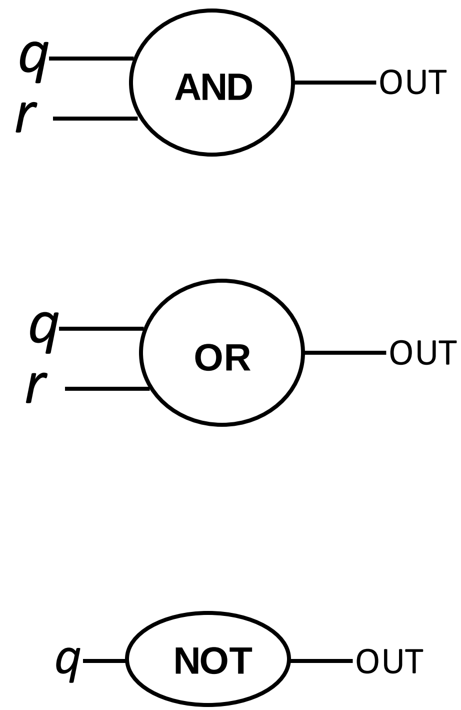
q	$\neg q$
T	F
F	T

q	OUT
1	0
0	1

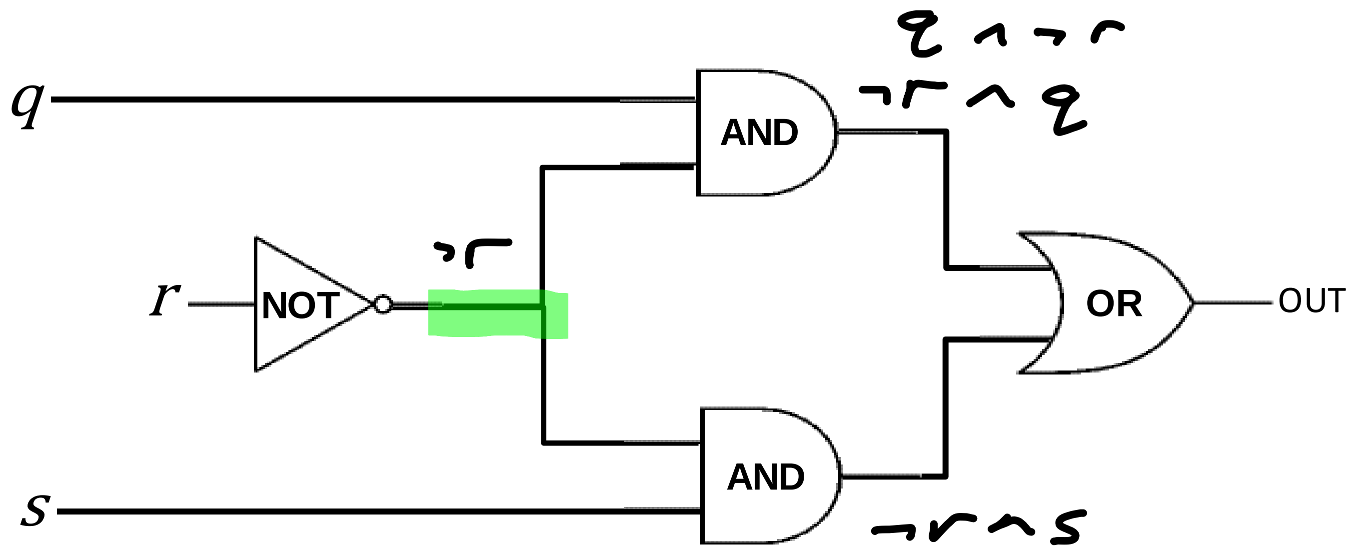


Blobs are Okay!

You may write gates using blobs instead of shapes!



Combinational Logic Circuits

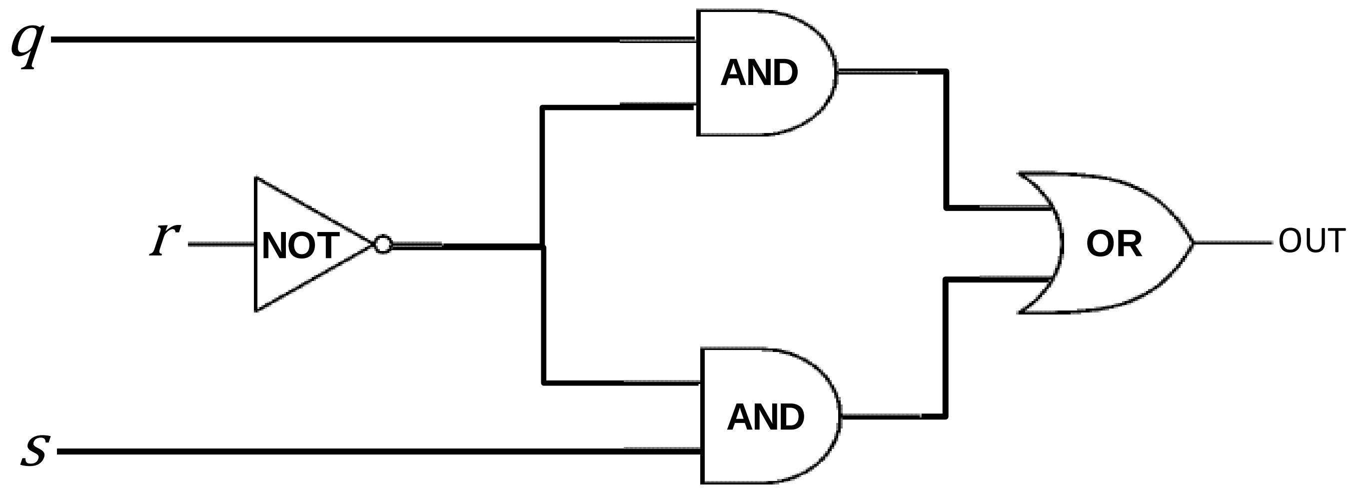


Wires can send one value to multiple gates!

$$(q \wedge \neg r) \vee (\neg r \wedge s)$$

$$\equiv \neg r \vee (q \wedge s)\ ?$$

Combinational Logic Circuits



Wires can send one value to multiple gates!

$$(q \land \neg r) \lor (\neg r \land s)$$

Computing Equivalence

Describe an algorithm for computing if two logical expressions/circuits are equivalent.

What is the run time of the algorithm?

Compute the entire truth table for both of them!

There are 2^n entries in the column for *n* variables.

for every truth assignment x:
check $A(x) \stackrel{?}{=} B(x)$

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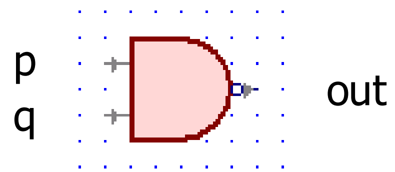
Other Useful Gates

NAND

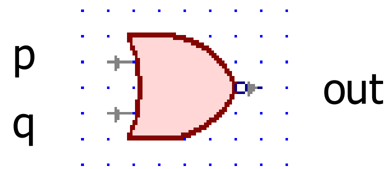
$\neg(p \wedge q)$



p	q	out
0	0	1
0	1	1
1	0	1
1	1	0

NOR

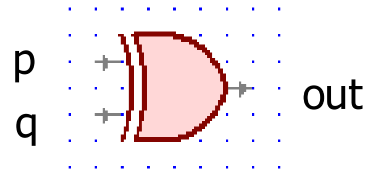
$\neg(p \vee q)$



p	q	out
0	0	1
0	1	0
1	0	0
1	1	0

XOR

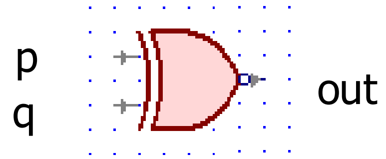
$p \oplus q$



p	q	out
0	0	0
0	1	1
1	0	1
1	1	0

XNOR

$p \leftrightarrow q$



p	q	out
0	0	1
0	1	0
1	0	0
1	1	1