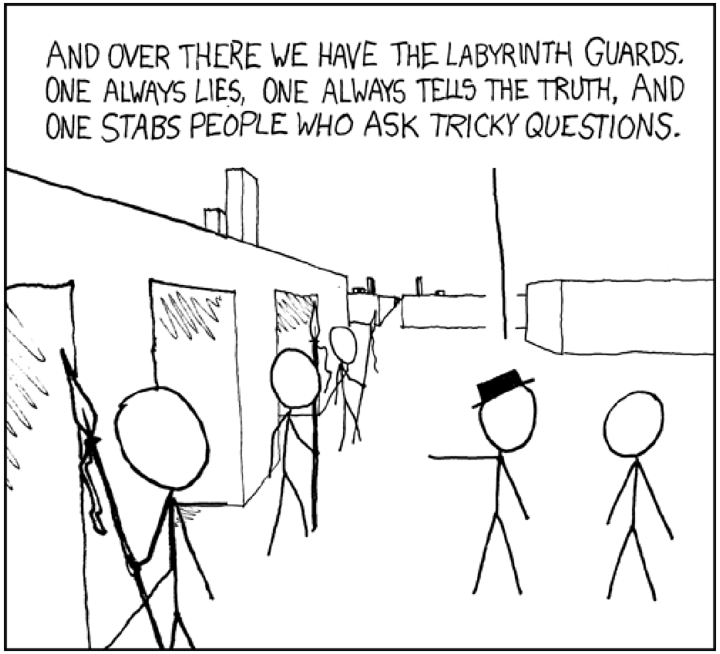
CSE 311: Foundations of Computing

Lecture 3: Digital Circuits & Equivalence



Recap from last class

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \to r \equiv \neg q \vee r$

Proving equivalence

- **Identity**
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- **Domination**
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  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
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  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
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  - $q \rightarrow r \equiv \neg q \vee r$

One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

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  - $q \rightarrow r \equiv \neg q \vee r$

One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

Example: Show that $\neg p \vee (p \vee p) \equiv T$

$$\neg p \vee (p \vee p) \equiv (\qquad\qquad\qquad)$$
$$\equiv (\qquad\qquad\qquad)$$
$$\equiv \mathbf{T}$$

Proving equivalence

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$$\neg p \vee (p \vee p) \equiv (\quad \neg p \vee p \quad) \quad \text{Idempotent}$$

$$\equiv (\qquad\qquad)$$

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One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

Example: Show that $\neg p \vee (p \vee p) \equiv T$

$\neg p \vee (p \vee p) \equiv (\neg p \vee p)$ Idempotent

$\equiv (p \vee \neg p)$ Commutative

$\equiv$ **T**

Proving equivalence

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$\equiv$ **T** Negation

What is a proof?

A proof is a logical argument that guarantees the conclusion is true. In this case, the conclusion is

$$\neg p \vee (p \vee p) \equiv \mathsf{T}$$

$\neg p \vee (p \vee p) \equiv (\neg p \vee p)$		Idempotent
	$\equiv (p \vee \neg p)$	Commutative
	$\equiv \mathbf{T}$	Negation

What is a proof?

A proof is a logical argument that guarantees the conclusion is true. In this case, the conclusion is

$$\neg p \vee (p \vee p) \equiv \mathsf{T}$$

The syntax there is a little terse. In full, it means:

(1) $\neg p \vee (p \vee p) \equiv \neg p \vee p$ by the Idempotent rule,

(2) $\neg p \vee p \equiv p \vee \neg p$ by the Commutative rule, and

(3) $p \vee \neg p \equiv \mathsf{T}$ by the Negation rule.

Therefore, we conclude $\neg p \vee (p \vee p) \equiv \mathsf{T}$

$\neg p \vee (p \vee p)$	$\equiv (\quad \neg p \vee p \quad)$	Idempotent
	$\equiv (\quad p \vee \neg p \quad)$	Commutative
	$\equiv \mathbf{T}$	Negation

Analyzing the Garfield Sentence with a Truth Table

Why not just use a truth table?

q	r	s	$\neg s$	$r \vee \neg s$	$r \wedge s$	$(r \wedge s) \rightarrow q$	$((r \wedge s) \rightarrow q) \wedge (r \vee \neg s)$
F	F	F	T	T	F	T	T
F	F	T	F	F	F	T	F
F	T	F	T	T	F	T	T
F	T	T	F	T	T	F	F
T	F	F	T	T	F	T	T
T	F	T	F	F	F	T	F
T	T	F	T	T	F	T	T
T	T	T	F	T	T	T	T

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

q	r	$q \rightarrow r$
T	T	T
T	F	F
F	T	T
F	F	T

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

q	***r***	***q→r***
T	T	T
T	F	F
F	T	T
F	F	T

$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv \neg q \vee r$

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Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$

$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$ **Associative**

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv \neg q \vee r$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

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  - $q \vee (q \wedge r) \equiv q$
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- **Law of implication**
  - $q \to r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$
$$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)] \quad \textbf{Associative}$$
$$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)] \quad \textbf{Distributive}$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv \neg q \vee r$$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

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- **Commutative**
  - $q \vee r \equiv r \vee q$
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  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
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  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
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  - $q \vee \neg q \equiv T$
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  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \to r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$

$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$ **Associative**

$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$ **Distributive**

$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}]$ **Negation**

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv \neg q \vee r$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

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  - $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$

$$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)] \quad \textbf{Associative}$$

$$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)] \quad \textbf{Distributive}$$

$$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}] \quad \textbf{Negation}$$

$$\equiv (q \wedge r) \vee [\neg q] \quad \textbf{Identity}$$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

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Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$	$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$	**Associative**
	$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$	**Distributive**
	$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}]$	**Negation**
	$\equiv (q \wedge r) \vee [\neg q]$	**Identity**
q no longer matters in $q \wedge r$ if $\neg q$ automatically makes the expression true.	$\equiv [\neg q] \vee (q \wedge r)$	**Commutative**
	$\equiv$	
	$\equiv$	
	$\equiv$	
	$\equiv$	
	$\equiv \neg q \vee r$	

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$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}]$ **Negation**

$\equiv (q \wedge r) \vee [\neg q]$ **Identity**

$\equiv [\neg q] \vee (q \wedge r)$ **Commutative**

$\equiv (\neg q \vee q) \wedge (\neg q \vee r)$ **Distributive**

$\equiv$

$\equiv$

$\equiv$

$\equiv \neg q \vee r$

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q no longer matters in $q \wedge r$ if $\neg q$ automatically makes the expression true.

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  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

$$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$$
$$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)] \quad \textbf{Associative}$$
$$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)] \quad \textbf{Distributive}$$
$$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}] \quad \textbf{Negation}$$
$$\equiv (q \wedge r) \vee [\neg q] \quad \textbf{Identity}$$
$$\equiv [\neg q] \vee (q \wedge r) \quad \textbf{Commutative}$$
$$\equiv (\neg q \vee q) \wedge (\neg q \vee r) \quad \textbf{Distributive}$$
$$\equiv (q \vee \neg q) \wedge (\neg q \vee r) \quad \textbf{Commutative}$$
$$\equiv$$
$$\equiv$$
$$\equiv \neg q \vee r$$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

q no longer matters in $q \wedge r$ if $\neg q$ automatically makes the expression true.

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \to r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

	$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$	
The last two terms are "vacuous truth" maybe the simplify to $\neg q$	$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$	Associative
	$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$	Distributive
	$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}]$	Negation
	$\equiv (q \wedge r) \vee [\neg q]$	Identity
q no longer matters in $q \wedge r$ if $\neg q$ automatically makes the expression true.	$\equiv [\neg q] \vee (q \wedge r)$	Commutative
	$\equiv (\neg q \vee q) \wedge (\neg q \vee r)$	Distributive
	$\equiv (q \vee \neg q) \wedge (\neg q \vee r)$	Commutative
	$\equiv \mathrm{T} \wedge (\neg q \vee r)$	Negation
	$\equiv$	
	$\equiv \neg q \vee r$	

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

A more complex equivalence proof

Show that $(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$

$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r)$

$\equiv (q \land r) \lor [(\neg q \land r) \lor (\neg q \land \neg r)]$ **Associative**

$\equiv (q \land r) \lor [\neg q \land (r \lor \neg r)]$ **Distributive**

$\equiv (q \land r) \lor [\neg q \land \mathrm{T}]$ **Negation**

$\equiv (q \land r) \lor [\neg q]$ **Identity**

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

$\equiv [\neg q] \lor (q \land r)$ **Commutative**

$\equiv (\neg q \lor q) \land (\neg q \lor r)$ **Distributive**

$\equiv (q \lor \neg q) \land (\neg q \lor r)$ **Commutative**

$\equiv \mathrm{T} \land (\neg q \lor r)$ **Negation**

$\equiv (\neg q \lor r) \land \mathrm{T}$ **Commutative**

$\equiv \neg q \lor r$

q no longer matters in $q \land r$ if $\neg q$ automatically makes the expression true.

- **Identity**
- $q \land T \equiv q$
- $q \lor F \equiv q$
- **Domination**
- $q \lor T \equiv T$
- $q \land F \equiv F$
- **Idempotent**
- $q \lor q \equiv q$
- $q \land q \equiv q$
- **Commutative**
- $q \lor r \equiv r \lor q$
- $q \land r \equiv r \land q$
- **De Morgan Laws**
- $\neg(q \land r) \equiv \neg q \lor \neg r$
- $\neg(q \lor r) \equiv \neg q \land \neg r$
- **Associative**
- $(q \lor r) \lor s \equiv q \lor (r \lor s)$
- $(q \land r) \land s \equiv q \land (r \land s)$
- **Distributive**
- $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$
- $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$
- **Absorption**
- $q \lor (q \land r) \equiv q$
- $q \land (q \lor r) \equiv q$
- **Negation**
- $q \lor \neg q \equiv T$
- $q \land \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \to r \equiv \neg q \lor r$

A more complex equivalence proof

Show that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$

	$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$	
The last two terms are "vacuous truth" maybe the simplify to $\neg q$	$\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$	**Associative**
	$\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$	**Distributive**
	$\equiv (q \wedge r) \vee [\neg q \wedge \mathrm{T}]$	**Negation**
	$\equiv (q \wedge r) \vee [\neg q]$	**Identity**
q no longer matters in $q \wedge r$ if $\neg q$ automatically makes the expression true.	$\equiv [\neg q] \vee (q \wedge r)$	**Commutative**
	$\equiv (\neg q \vee q) \wedge (\neg q \vee r)$	**Distributive**
	$\equiv (q \vee \neg q) \wedge (\neg q \vee r)$	**Commutative**
	$\equiv \mathrm{T} \wedge (\neg q \vee r)$	**Negation**
	$\equiv (\neg q \vee r) \wedge \mathrm{T}$	**Commutative**
	$\equiv \neg q \vee r$	**Identity**

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

Prove this is a Tautology

- **Associative**
- $(q \vee r) \vee s \equiv q \vee (r \vee s)$
- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
- $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
- $q \vee (q \wedge r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- **Negation**
- $q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \to r \equiv \neg q \vee r$

$$(q \wedge r) \to (r \vee q)$$

- **Identity**
- $q \wedge T \equiv q$
- $q \vee F \equiv q$
- **Domination**
- $q \vee T \equiv T$
- $q \wedge F \equiv F$
- **Idempotent**
- $q \vee q \equiv q$
- $q \wedge q \equiv q$
- **Commutative**
- $q \vee r \equiv r \vee q$
- $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
- $\neg(q \wedge r) \equiv \neg q \vee \neg r$
- $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Prove this is a Tautology

$$(q \wedge r) \rightarrow (r \vee q)$$

Use a series of equivalences:

$$(q \wedge r) \rightarrow (r \vee q) \equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv \quad \mathbf{T}$$

- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace $\rightarrow$; move $\neg$ inside; simplify

Prove this is a Tautology

- **Associative**
- $(q \lor r) \lor s \equiv q \lor (r \lor s)$
- $(q \land r) \land s \equiv q \land (r \land s)$
- **Distributive**
- $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$
- $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$
- **Absorption**
- $q \lor (q \land r) \equiv q$
- $q \land (q \lor r) \equiv q$
- **Negation**
- $q \lor \neg q \equiv T$
- $q \land \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \to r \equiv \neg q \lor r$

$$(q \land r) \to (r \lor q)$$

Use a series of equivalences:

$$(q \land r) \to (r \lor q) \equiv \neg(q \land r) \lor (r \lor q)$$ Law of Implication

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$ **T**

- **Identity**
- $q \land T \equiv q$
- $q \lor F \equiv q$
- **Domination**
- $q \lor T \equiv T$
- $q \land F \equiv F$
- **Idempotent**
- $q \lor q \equiv q$
- $q \land q \equiv q$
- **Commutative**
- $q \lor r \equiv r \lor q$
- $q \land r \equiv r \land q$
- **De Morgan Laws**
- $\neg(q \land r) \equiv \neg q \lor \neg r$
- $\neg(q \lor r) \equiv \neg q \land \neg r$

Our strategy: Replace →; move ¬ inside; simplify

Prove this is a Tautology

$$(q \wedge r) \rightarrow (r \vee q)$$

Use a series of equivalences:

$$(q \wedge r) \rightarrow (r \vee q) \equiv \neg(q \wedge r) \vee (r \vee q)$$ Law of Implication

$$\equiv (\neg q \vee \neg r) \vee (r \vee q)$$ DeMorgan

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$ **T**

Our strategy: Replace $\rightarrow$; move $\neg$ inside; simplify

- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Prove this is a Tautology

- **Associative**
- $(q \vee r) \vee s \equiv q \vee (r \vee s)$
- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
- $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
- $q \vee (q \wedge r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- **Negation**
- $q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \rightarrow r \equiv \neg q \vee r$

$$(q \wedge r) \rightarrow (r \vee q)$$

Use a series of equivalences:

$$(q \wedge r) \rightarrow (r \vee q) \equiv \neg(q \wedge r) \vee (r \vee q)$$ Law of Implication

$$\equiv (\neg q \vee \neg r) \vee (r \vee q)$$ DeMorgan

$$\equiv \neg q \vee (\neg r \vee (r \vee q))$$ Associative

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$ **T**

- **Identity**
- $q \wedge T \equiv q$
- $q \vee F \equiv q$
- **Domination**
- $q \vee T \equiv T$
- $q \wedge F \equiv F$
- **Idempotent**
- $q \vee q \equiv q$
- $q \wedge q \equiv q$
- **Commutative**
- $q \vee r \equiv r \vee q$
- $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
- $\neg(q \wedge r) \equiv \neg q \vee \neg r$
- $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace $\rightarrow$; move $\neg$ inside; simplify

Prove this is a Tautology

- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- **Negation**
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \rightarrow r \equiv \neg q \vee r$

$$(q \wedge r) \rightarrow (r \vee q)$$

Use a series of equivalences:

$$(q \wedge r) \rightarrow (r \vee q) \equiv \neg(q \wedge r) \vee (r \vee q) \quad \text{Law of Implication}$$

$$\equiv (\neg q \vee \neg r) \vee (r \vee q) \quad \text{DeMorgan}$$

$$\equiv \neg q \vee (\neg r \vee (r \vee q)) \quad \text{Associative}$$

$$\equiv \neg q \vee ((\neg r \vee r) \vee q) \quad \text{Associative}$$

$$\equiv$$

$$\equiv$$

$$\equiv$$

$$\equiv$$

$$\equiv \quad \mathbf{T}$$

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace →; move ¬ inside; simplify

Prove this is a Tautology

- **Associative**
  - $(q \lor r) \lor s \equiv q \lor (r \lor s)$
  - $(q \land r) \land s \equiv q \land (r \land s)$
- **Distributive**
  - $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$
  - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$
- **Absorption**
  - $q \lor (q \land r) \equiv q$
  - $q \land (q \lor r) \equiv q$
- **Negation**
  - $q \lor \neg q \equiv T$
  - $q \land \neg q \equiv F$
- **Double negation**
  - $\neg(\neg q) \equiv q$
- **Law of implication**
  - $q \to r \equiv \neg q \lor r$

$$(q \land r) \to (r \lor q)$$

Use a series of equivalences:

$(q \land r) \to (r \lor q) \equiv \neg(q \land r) \lor (r \lor q)$	Law of Implication
$\equiv (\neg q \lor \neg r) \lor (r \lor q)$	DeMorgan
$\equiv \neg q \lor (\neg r \lor (r \lor q))$	Associative
$\equiv \neg q \lor ((\neg r \lor r) \lor q)$	Associative
$\equiv \neg q \lor (q \lor (\neg r \lor r))$	Commutative
$\equiv$	
$\equiv$	
$\equiv$	
$\equiv$ **T**	

- **Identity**
  - $q \land T \equiv q$
  - $q \lor F \equiv q$
- **Domination**
  - $q \lor T \equiv T$
  - $q \land F \equiv F$
- **Idempotent**
  - $q \lor q \equiv q$
  - $q \land q \equiv q$
- **Commutative**
  - $q \lor r \equiv r \lor q$
  - $q \land r \equiv r \land q$
- **De Morgan Laws**
  - $\neg(q \land r) \equiv \neg q \lor \neg r$
  - $\neg(q \lor r) \equiv \neg q \land \neg r$

Our strategy: Replace $\to$; move $\neg$ inside; simplify

Prove this is a Tautology

- **Associative**
- $(q \lor r) \lor s \equiv q \lor (r \lor s)$
- $(q \land r) \land s \equiv q \land (r \land s)$
- **Distributive**
- $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$
- $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$
- **Absorption**
- $q \lor (q \land r) \equiv q$
- $q \land (q \lor r) \equiv q$
- **Negation**
- $q \lor \neg q \equiv T$
- $q \land \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \to r \equiv \neg q \lor r$

$$(q \land r) \to (r \lor q)$$

Use a series of equivalences:

$(q \land r) \to (r \lor q)$	$\equiv \neg(q \land r) \lor (r \lor q)$	Law of Implication
	$\equiv (\neg q \lor \neg r) \lor (r \lor q)$	DeMorgan
	$\equiv \neg q \lor (\neg r \lor (r \lor q))$	Associative
	$\equiv \neg q \lor ((\neg r \lor r) \lor q)$	Associative
	$\equiv \neg q \lor (q \lor (\neg r \lor r))$	Commutative
	$\equiv (\neg q \lor q) \lor (\neg r \lor r)$	Associative
	$\equiv$	
	$\equiv$	
	$\equiv$ **T**	

- **Identity**
- $q \land T \equiv q$
- $q \lor F \equiv q$
- **Domination**
- $q \lor T \equiv T$
- $q \land F \equiv F$
- **Idempotent**
- $q \lor q \equiv q$
- $q \land q \equiv q$
- **Commutative**
- $q \lor r \equiv r \lor q$
- $q \land r \equiv r \land q$
- **De Morgan Laws**
- $\neg(q \land r) \equiv \neg q \lor \neg r$
- $\neg(q \lor r) \equiv \neg q \land \neg r$

Our strategy: Replace $\to$; move $\neg$ inside; simplify

Prove this is a Tautology

- **Associative**
- $(q \vee r) \vee s \equiv q \vee (r \vee s)$
- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
- $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- **Absorption**
- $q \vee (q \wedge r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- **Negation**
- $q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- **Double negation**
- $\neg(\neg q) \equiv q$
- **Law of implication**
- $q \rightarrow r \equiv \neg q \vee r$

$$(q \wedge r) \rightarrow (r \vee q)$$

Use a series of equivalences:

$(q \wedge r) \rightarrow (r \vee q) \equiv \neg(q \wedge r) \vee (r \vee q)$	Law of Implication
$\equiv (\neg q \vee \neg r) \vee (r \vee q)$	DeMorgan
$\equiv \neg q \vee (\neg r \vee (r \vee q))$	Associative
$\equiv \neg q \vee ((\neg r \vee r) \vee q)$	Associative
$\equiv \neg q \vee (q \vee (\neg r \vee r))$	Commutative
$\equiv (\neg q \vee q) \vee (\neg r \vee r)$	Associative
$\equiv (q \vee \neg q) \vee (r \vee \neg r)$	Commutative (twice)
$\equiv$	
$\equiv$ **T**	

- **Identity**
- $q \wedge T \equiv q$
- $q \vee F \equiv q$
- **Domination**
- $q \vee T \equiv T$
- $q \wedge F \equiv F$
- **Idempotent**
- $q \vee q \equiv q$
- $q \wedge q \equiv q$
- **Commutative**
- $q \vee r \equiv r \vee q$
- $q \wedge r \equiv r \wedge q$
- **De Morgan Laws**
- $\neg(q \wedge r) \equiv \neg q \vee \neg r$
- $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace $\rightarrow$; move $\neg$ inside; simplify

Prove this is a Tautology

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$\equiv \neg q \vee (q \vee (\neg r \vee r))$	Commutative
$\equiv (\neg q \vee q) \vee (\neg r \vee r)$	Associative
$\equiv (q \vee \neg q) \vee (r \vee \neg r)$	Commutative (twice)
$\equiv \mathbf{T} \vee \mathbf{T}$	Negation (twice)
$\equiv \mathbf{T}$	

Our strategy: Replace →; move ¬ inside; simplify

- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- **Distributive**
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  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
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  - $q \vee (q \wedge r) \equiv q$
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- **Negation**
  - $q \vee \neg q \equiv T$
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  - $\neg(\neg q) \equiv q$
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  - $q \wedge F \equiv F$
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  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
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  - $q \vee r \equiv r \vee q$
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  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Prove this is a Tautology

- **Associative**
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
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$\equiv (q \vee \neg q) \vee (r \vee \neg r)$	Commutative (twice)
$\equiv \mathbf{T} \vee \mathbf{T}$	Negation (twice)
$\equiv \mathbf{T}$	Domination/Identity

- **Identity**
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- **Domination**
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- **Idempotent**
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- **Commutative**
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
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  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$

Our strategy: Replace $\rightarrow$; move $\neg$ inside; simplify

Logical Proofs of Equivalence/Tautology

- Not smaller than truth tables when there are only a few propositional variables...

- ...but usually ***much shorter*** than truth table proofs when there are many propositional variables

- A big advantage will be that we can extend them to a more in-depth understanding of logic for which truth tables don't apply.

Lecture 3 Activity

- You will be assigned to **breakout rooms**. Please:
- Introduce yourself
- Choose someone to share screen, showing this PDF
- Show that $p \to q \equiv \neg q \to \neg p$ using a sequence of elementary equivalences.

Fill out a poll everywhere for Activity Credit!
Go to pollev.com/philipmg and login with your UW identity

- Identity
  - $q \wedge T \equiv q$
  - $q \vee F \equiv q$
- Domination
  - $q \vee T \equiv T$
  - $q \wedge F \equiv F$
- Idempotent
  - $q \vee q \equiv q$
  - $q \wedge q \equiv q$
- Commutative
  - $q \vee r \equiv r \vee q$
  - $q \wedge r \equiv r \wedge q$
- De Morgan Laws
  - $\neg(q \wedge r) \equiv \neg q \vee \neg r$
  - $\neg(q \vee r) \equiv \neg q \wedge \neg r$
- Associative
  - $(q \vee r) \vee s \equiv q \vee (r \vee s)$
  - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- Distributive
  - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
  - $q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$
- Absorption
  - $q \vee (q \wedge r) \equiv q$
  - $q \wedge (q \vee r) \equiv q$
- Negation
  - $q \vee \neg q \equiv T$
  - $q \wedge \neg q \equiv F$
- Double negation
  - $\neg(\neg q) \equiv q$
- Law of implication
  - $q \to r \equiv \neg q \vee r$

Digital Circuits

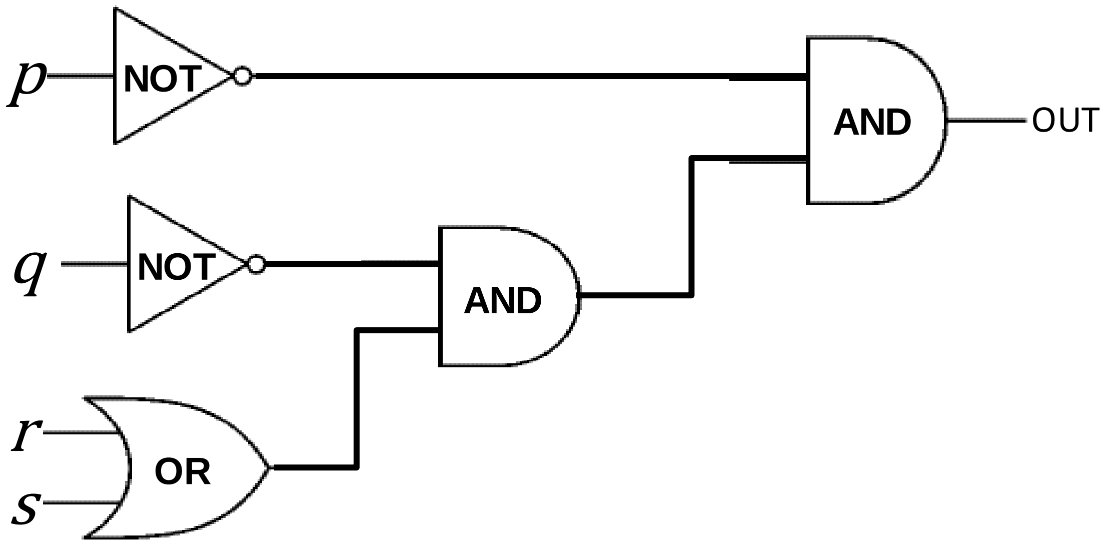
Computing With Logic

- T corresponds to 1 or "high" voltage
- F corresponds to 0 or "low" voltage

Gates

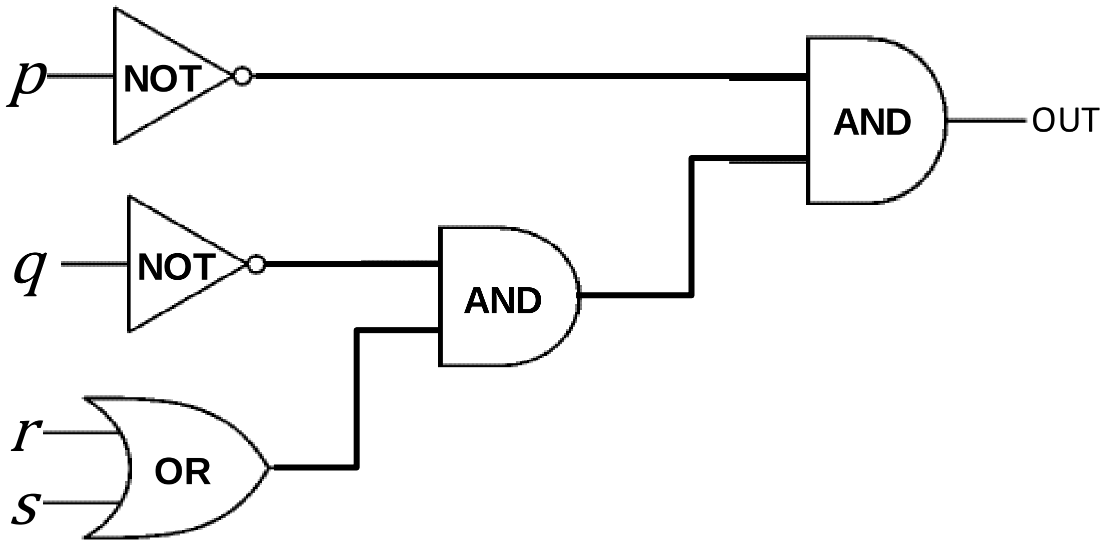
- Take inputs and produce outputs (functions)
- Several kinds of gates
- Correspond to propositional connectives (most of them)

Combinational Logic Circuits



Values get sent along wires connecting gates

Combinational Logic Circuits



Values get sent along wires connecting gates

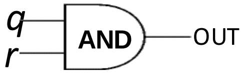
$$\neg p \land (\neg q \land (r \lor s))$$

And Gate

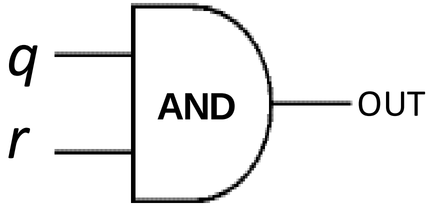
AND Connective vs. AND Gate

$q \wedge r$

q	r	$q \wedge r$
T	T	T
T	F	F
F	T	F
F	F	F



q	r	OUT
1	1	1
1	0	0
0	1	0
0	0	0



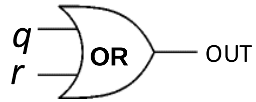
"block looks like D of AND"

Or Gate

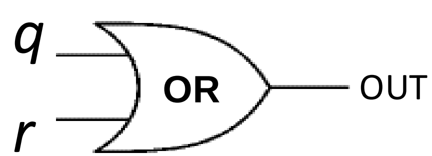
OR Connective vs. OR Gate

q ∨ *r*

q	*r*	*q* ∨ *r*
T	T	T
T	F	T
F	T	T
F	F	F



q	*r*	OUT
1	1	1
1	0	1
0	1	1
0	0	0

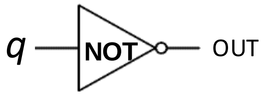


“arrowhead block looks like V”

Not Gates

NOT Connective vs. **NOT Gate**

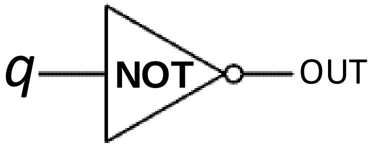
$\neg q$



Also called *inverter*

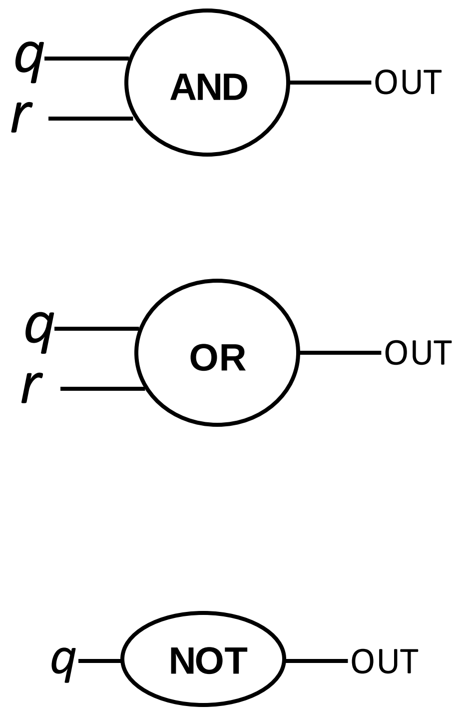
q	$\neg q$
T	F
F	T

q	OUT
1	0
0	1

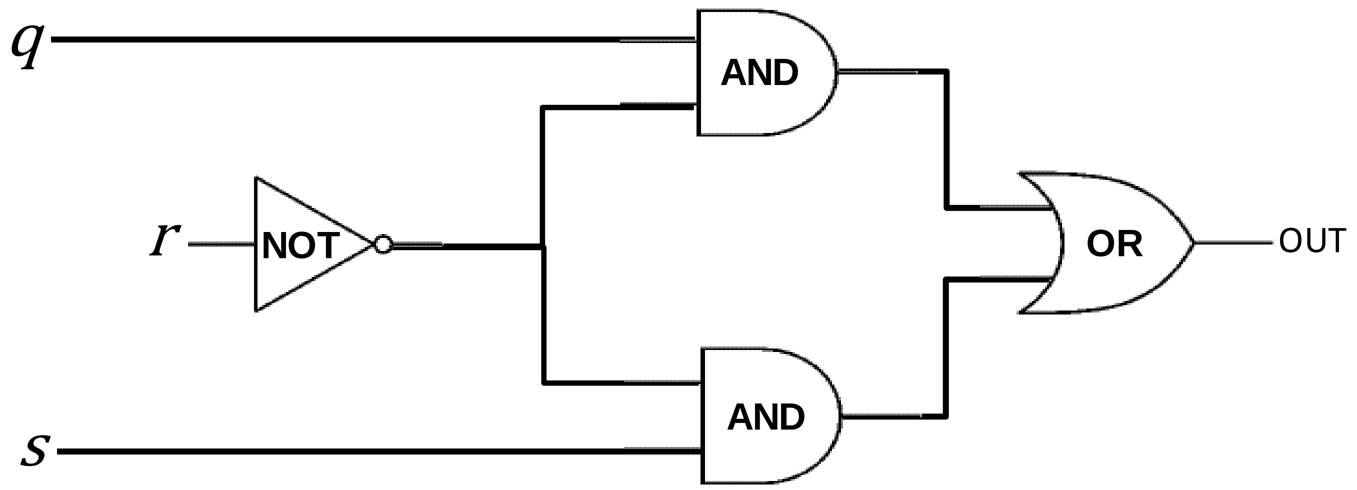


Blobs are Okay!

You may write gates using blobs instead of shapes!

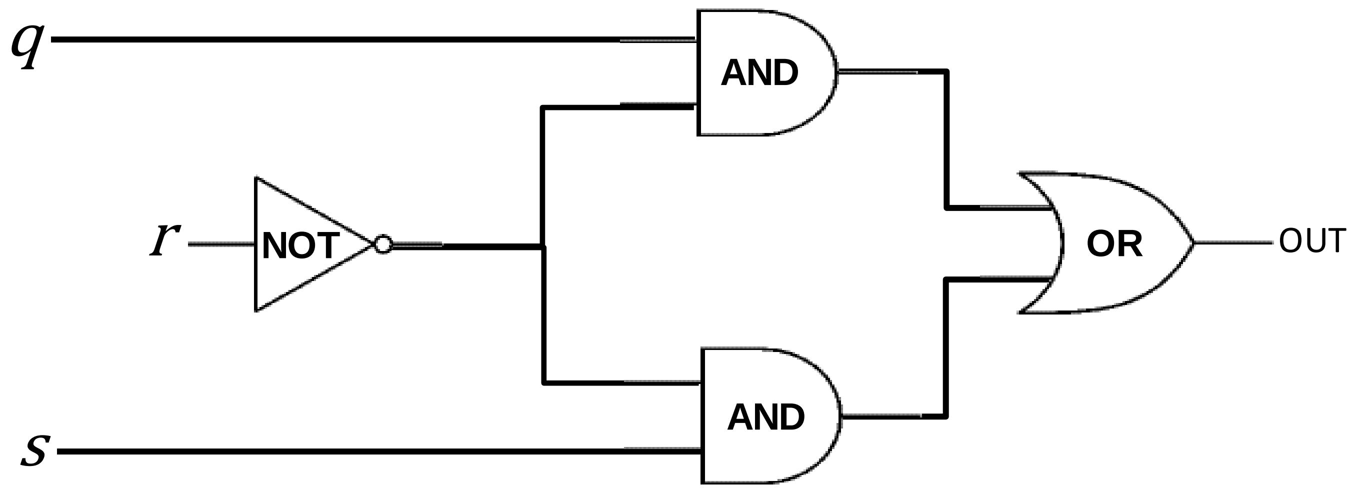


Combinational Logic Circuits



Wires can send one value to multiple gates!

Combinational Logic Circuits



Wires can send one value to multiple gates!

$$(q \wedge \neg r) \vee (\neg r \wedge s)$$

Computing Equivalence

Describe an algorithm for computing if two logical expressions/circuits are equivalent.

What is the run time of the algorithm?

Compute the entire truth table for both of them!

There are 2^n entries in the column for *n* variables.

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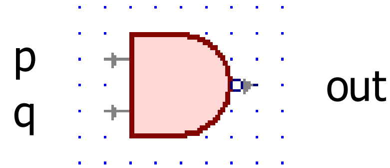
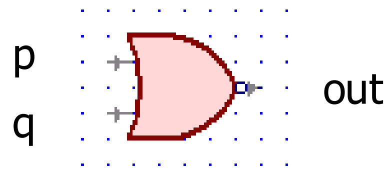
Other Useful Gates

NAND
$\neg(p \land q)$



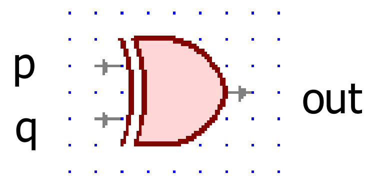
p	q	out
0	0	1
0	1	1
1	0	1
1	1	0

NOR
$\neg(p \lor q)$



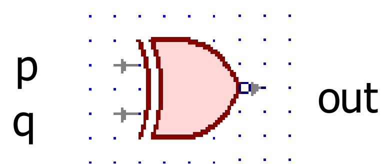
p	q	out
0	0	1
0	1	0
1	0	0
1	1	0

XOR
$p \oplus q$



p	q	out
0	0	0
0	1	1
1	0	1
1	1	0

XNOR
$p \leftrightarrow q$



p	q	out
0	0	1
0	1	0
1	0	0
1	1	1