Homework 1: Propositional Logic

Due date: Friday January 15 at 11:59 PM (Seattle time, i.e. GMT-8)
If you work with others (and you should!), remember to follow the collaboration policy.
In general, you are graded on both the clarity and accuracy of your work. Your solution should be clear enough that someone in the class who had not seen the problem before would understand it.
We sometimes describe approximately how long our explanations are. These are intended to help you understand approximately how much detail we are expecting.

Be sure to read the grading guidelines for more information on what we're looking for.

1. Translation [16 points]

Translate the English statements into symbolic logic, making the atomic propositions as basic as possible.

(a) To watch Tangled, it suffices to have a DisneyPlus account or a DVD. [4 points]
Note: propositions should be able to have a truth value, so "watch Tangled" is not a proposition, but "One can watch Tangled" is, for example.

(b) The file can be created only if the disk is not full and the user has write permissions. [4 points]

(c) Define a set of *at most three* atomic propositions. Then, use them to translate all of these sentences about why dogs lick you into logic. [8 points]

(i) If the dog licks you, then they show you affection.

(ii) Unless they want attention, the dog does not lick you.

(iii) The dog licks you only if they show you affection or want attention.

2. Trickier translation [10 points]

The following sentences are idiomatic in English...but not very clear logically. Convert each statement into propositional logic, then write an English sentence that has the same meaning, but is clearer logicially.

(a) If you leave a room, you should turn the light off, unless there is someone in the room or you don't see the lightswitch.

(b) I can go home, if the lightrail is running, and if my lightrail pass is active as well.

3. Inequivalence [16 points]

For each part, find a truth assignment (i.e. an assignment of `True` or `False` to $p, q,$ and r) to show the pair of statements are not equivalent. Explain why your assignments work (our explanations are 1-2 sentences).

(a) $(q \to p) \to r$ vs. $q \to (r \to p)$.

(b) $p \vee (p \wedge q)$ vs. $p \vee q$.

(c) $p \to (q \wedge r)$ vs. $(p \to q) \wedge r$

(d) $(p \wedge q) \to r$ vs. $(\neg p \wedge \neg q) \vee r$

4. Compound Proposition [8 points]

Find a compound proposition involving the variables $p, q, r,$ and s that is true precisely when **at least** two of $p, q, r,$ and s are true. Explain why your answer works (1-2 sentences). *Note: By "precisely," we mean also that the proposition should be false whenever the condition is not met.*

5. Proof [24 points]

In Lecture 3 (and in Lecture 2 slide 62) we gave a symbolic proof that $(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv (\neg p \vee q)$. In this problem we will give another proof.

(a) Our intuition for the proof in class was "the last two pieces of the formula correspond to vacuous truth." Identify a commonality in the first two pieces of the formula, and describe it.
(Your description should be similar in spirit to the one from class, but you don't need to use fancy vocabulary like "vacuous truth" – our answer here is one sentence) [4 points]

(b) Give another proof of the formula that matches the intuition from part a instead of the intuition from class. [16 points]
Read the symbolic proof guidelines before you start.
Hint: your proof, if it matches your intuition from (a) will be different from the one from class – at least some of the intermediate expressions will be different.

(c) In class we labeled portions of the proof in purple with high-level descriptions of what they are doing (lecture 2 slide 62, left side). Produce similar labels for your proof. Submit your answer in the form "Steps [X] to [Y]: [label]" for each part. [4 points]
Note: The goal here is to give intuition for what is happening at a higher level than individual steps.

6. Highly Illogical [12 points]

This problem is inspired by the Star Trek universe. A "transporter" is a teleportation machine. Food is "replicated" instead of taken out of the freezer. "Ice cream" is still delicious.

As a result of a tragic transporter accident, you have been named the legal guardian of T'Vin, a young Vulcan child. T'Vin is ruthlessly and perfectly logical – he takes each statement at its logical meaning; he does not believe you will lie, but he does not accept unstated intentions.

You want T'Vin to clean his room, and (like many parents) you've resorted to bribery – you are willing to replicate some ice cream in exchange for compliance. T'Vin loves ice cream and hates cleaning. He would rather have ice cream and clean his room than do neither, but will not clean his room without the reward of ice cream, nor will he clean if he has any hope of getting ice cream some other way.

(a) You tell T'Vin "If you don't clean your room, then I won't replicate you any ice cream."
He looks at you and says "It is not yet logical to clean my room." Why will he not comply? (1-2 sentences)

(b) You try again: "Forget the first promise" you say "If you clean your room, then I will replicate you some ice cream." Still he says, "It is not yet logical to clean my room" Why is he not cleaning? (1-2 sentences)

(c) Give a logical sentence (or sentences) which will compel T'Vin to clean his room. Argue that T'Vin will finally clean his room. (2-4 sentences)