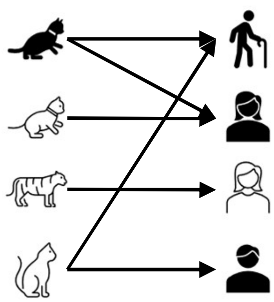
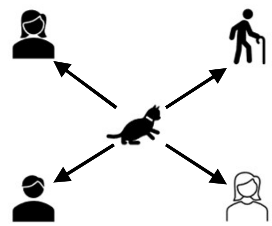
Try it yourselves

Every cat loves some human.



There is a cat that loves every human.



Let your domain of discourse be mammals.
Use the predicates `Cat`(x), `Dog`(x), and `Loves`(x, y) to mean x loves y.

Try it!

Given: $p \vee q$, $(r \wedge s) \rightarrow \neg q$, r.
Show: $s \rightarrow p$

Eliminate ∧
$$\frac{A \wedge B}{\therefore \quad A, B}$$

Eliminate ∨
$$\frac{A \vee B, \neg A}{\therefore \quad B}$$

Intro ∧
$$\frac{A; B}{\therefore \quad A \wedge B}$$

Intro ∨
$$\frac{A}{\therefore A \vee B, B \vee A}$$

Direct Proof rule
$$\frac{A \Rightarrow B}{A \rightarrow B}$$

Modus Ponens
$$\frac{P \rightarrow Q; P}{\therefore \quad Q}$$

You can still use all the propositional logic equivalences too!

Proof Using Quantifiers

Suppose we know $\exists x P(x)$ and $\forall y[\, P(y) \rightarrow Q(y)]$. Conclude $\exists x Q(x)$.

Intro ∃: $\dfrac{P(c) \text{ for some } c}{\therefore\ \exists x\, P(x)}$

Eliminate ∃: $\dfrac{\exists x P(x)}{\therefore\ P(c) \text{ for a fresh } c}$

Eliminate ∀: $\dfrac{\forall x\, P(x)}{\therefore\ P(a) \text{ for any } a}$

Intro ∀: $\dfrac{P(a);\, a \text{ is arbitrary}}{\therefore\ \forall x\, P(x)}$

Find The Bug

Let your domain of discourse be integers.
We claim that given $\forall x \exists y\, \texttt{Greater}(y, x)$, we can conclude $\exists y \forall x\, \texttt{Greater}(y, x)$
Where $\texttt{Greater}(y, x)$ means $y > x$

1. $\forall x \exists y\, \texttt{Greater}(y, x)$ — Given
2. Let a be an arbitrary integer — --
3. $\exists y\, \texttt{Greater}(y, a)$ — Elim ∀ (1)
4. $b \geq a$ — Elim ∃ (2)
5. $\forall x\, \texttt{Greater}(b, x)$ — Intro ∀ (4)
6. $\exists y \forall x\, \texttt{Greater}(y, x)$ — Intro ∃ (5)