Section 3

CSE 311 - Sp 2022

Administrivia

Announcements and Reminders

- HW2 due yesterday 10PM on Gradescope
  - Final late due date is Saturday 4/16 @ 10pm
- HW1 grades out now
  - Regrade requests are open for one week
  - If you think your work may have been graded incorrectly, please submit a regrade request!
- HW3 is out!
  - Due Wednesday 4/20 @ 10pm
- Check the course website for OH times!
  - Friday, we have extended Allie & Sandy's OH to be from 4:30-6:20, and we'll be offering some targeted help during the first hour in particular, going through some of the questions from the new HW assignment and focusing on understanding what the question is asking, where you should start, and where you are trying to go

More Practice with Predicates & Quantifiers

Predicates and Quantifiers Review

- Predicate: a function that outputs true or false
  - Cat(x) := “x is a cat”
  - LessThan(x, y) := “x < y
- Domain of Discourse: the types of inputs allowed in predicates
  - Numbers, mammals, cats and dogs, people in this class, etc.
- Quantifiers
  - Universal Quantifier ∀x: for all x, for every x
  - Existential Quantifier ∃x: there is an x, there exists an x, for some x
- Domain Restriction
  - Universal Quantifier ∀x: add a hypothesis to an implication
  - Existential Quantifier ∃x: there is an x, AND in the restriction

Problem 1 - Domain Restriction

Translate each of the following sentences into logical notation. These translations require some of our quantifier tricks. You may use the operators + and · which take two numbers as input and evaluate to their sum or product, respectively.

a) Domain: Positive integers; Predicates: *Even, Prime, Equal*
   "There is only one positive integer that is prime and even."
b) Domain: Real numbers; Predicates: *Even, Prime, Equal*
   "There are two different prime numbers that sum to an even number."
c) Domain: Real numbers; Predicates: *Even, Prime, Equal*
   "The product of two distinct prime numbers is not prime."
d) Domain: Real numbers; Predicates: *Even, Prime, Equal, Positive, Greater, Integer*
   "For every positive integer, there is a greater even integer"

Work on parts (a) and (b) with the people around you, and then we'll go over it together!

Problem 1 - Domain Restriction

a) Domain: Positive integers; Predicates: *Even, Prime, Equal*
   "There is only one positive integer that is prime and even."

Problem 1 - Domain Restriction

a) Domain: Positive integers; Predicates: *Even, Prime, Equal*
"There is only one positive integer that is prime and even."

We can start out with:

$\exists x(\text{Prime}(x) \land \text{Even}(x))$

Problem 1 - Domain Restriction

a) Domain: Positive integers; Predicates: *Even, Prime, Equal*
"There is only one positive integer that is prime and even."

We can start out with:

$\exists x(Prime(x) \land Even(x))$

But now we need to add in the restriction that this x is the ONLY positive integer that is prime and even. This is a technique you'll use whenever you need to have only one of something:

$\exists x(Prime(x) \land Even(x) \land \forall y[\neg Equal(x, y) \rightarrow \neg(Even(y) \land Prime(y))])$

Or, we could use the contrapositive:
$\exists x(Prime(x) \land Even(x) \land \forall y[(Even(y) \land Prime(y)) \rightarrow Equal(x, y)])$

Problem 1 - Domain Restriction

b) Domain: Real numbers; Predicates: *Even, Prime, Equal* “There are two different prime numbers that sum to an even number.”

Problem 1 - Domain Restriction

b) Domain: Real numbers; Predicates: *Even, Prime, Equal* "There are two different prime numbers that sum to an even number."

Seems like maybe we should be able to say something like:

$\exists x \exists y(\text{Prime}(x) \land \text{Prime}(y) \land \text{Even}(x + y))$

Problem 1 - Domain Restriction

b) Domain: Real numbers; Predicates: *Even, Prime, Equal* “There are two different prime numbers that sum to an even number.”

Seems like maybe we should be able to say something like:

$\exists x \exists y(Prime(x) \land Prime(y) \land Even(x + y))$

But this leaves open the possibility of x and y being equal (so they won’t be two DIFFERENT numbers). So, we need to explicitly add in that x and y are not equal:

$\exists x \exists y(Prime(x) \land Prime(y) \land Even(x + y) \land \neg Equal(x, y))$

Problem 2 - ctrl-z

Translate these logical expressions to English. For each of the translations, assume that domain restriction is being used and take that into account in your English versions.

Let your domain be all UW Students. Predicates 143Student(x) and 311Student(x) mean the student is in CSE 143 and 311, respectively. BioMajor(x) means x is a bio major, DidHomeworkOne(x) means the student did homework 1 (of 311). Finally KnowsJava(x) and KnowsDeMorgan(x) mean x knows Java and knows DeMorgan's Laws, respectively.

a) $\forall x(\text{143Student}(x) \rightarrow \text{KnowsJava}(x))$
b) $\exists x(\text{143Student}(x) \wedge \text{BioMajor}(x))$
c) $\forall x([\text{311Student}(x) \wedge \text{DidHomeworkOne}(x)] \rightarrow \text{KnowsDeMorgan}(x))$

Work on parts (a) and (c) with the people around you, and then we'll go over it together!

Problem 2 - ctrl-z

a) $\forall x(143Student(x) \rightarrow KnowsJava(x))$

Problem 2 - ctrl-z

a) $\forall x(143Student(x) \rightarrow KnowsJava(x))$

Every 143 student knows java.

"If a UW student is a 143 student, then they know java" is a valid translation of the original sentence, but it is not taking advantage of the domain restriction.

Problem 2 - ctrl-z

c) $\forall x([311Student(x) \land DidHomeworkOne(x)] \rightarrow KnowsDeMorgan(x))$

Problem 2 - ctrl-z

c) $\forall x([311Student(x) \wedge DidHomeworkOne(x)] \rightarrow KnowsDeMorgan(x))$

All 311 students who do Homework 1 know DeMorgan's Laws.

"If a UW student is a 311 student and they did Homework 1, then they know DeMorgan's Laws" is a valid translation of the original sentence, but it is not taking advantage of the domain restriction.

Problem 4 - Quantifier Switch

Consider the following pairs of sentences. For each pair, determine if one implies the other, if they are equivalent, or neither.

a) $\forall x \, \forall y \, P(x, y)$ $\qquad$ $\forall y \, \forall x \, P(x, y)$

b) $\exists x \, \exists y \, P(x, y)$ $\qquad$ $\exists y \, \exists x \, P(x, y)$

c) $\forall x \, \exists y \, P(x, y)$ $\qquad$ $\forall y \, \exists x \, P(x, y)$

d) $\forall x \, \exists y \, P(x, y)$ $\qquad$ $\exists x \, \forall y \, P(x, y)$

e) $\forall x \, \exists y \, P(x, y)$ $\qquad$ $\exists y \, \forall x \, P(x, y)$

Work on parts (d) and (e) with the people around you, and then we'll go over it together!

Problem 4 - Quantifier Switch

d) $\forall x \, \exists y \, P(x, y)$ $\exists x \, \forall y \, P(x, y)$

Problem 4 - Quantifier Switch

d) $\forall x \exists y P(x, y)$ $\exists x \forall y P(x, y)$

Different

For all x, there is a y vs. there exists an x that, for all y

Everyone likes someone vs. someone likes everyone

Problem 4 - Quantifier Switch

e) $\forall x \exists y P(x, y)$ $\exists y \forall x P(x, y)$

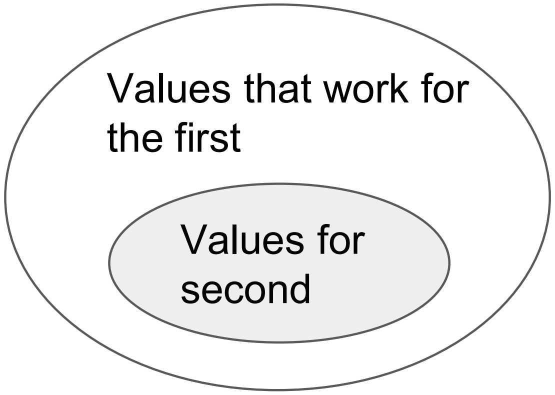
Problem 4 - Quantifier Switch

e) $\forall x \exists y\, P(x, y)$ $\exists y \forall x\, P(x, y)$

The second implies the first

For all x, there is a y, vs. there exists a y that, for all x

The second is stronger since a specific y must work for all x whereas the for the first, the y value does not have to be the same for every x



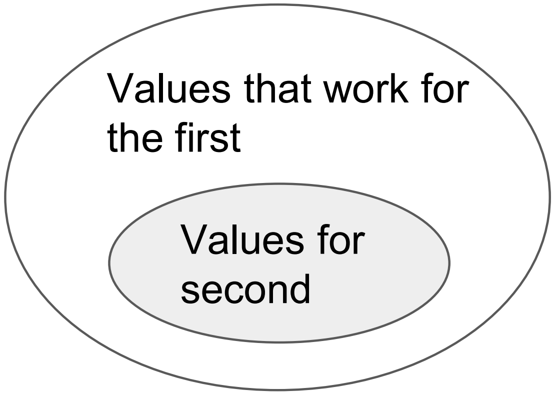
Problem 4 - Quantifier Switch

e) $\forall x \exists y P(x, y)$ $\exists y \forall x P(x, y)$

The second implies the first

For all x, there is a y, vs. there exists a y that, for all x

The second is stronger since a specific y must work for all x whereas the for the first, the y value does not have to be the same for every x



Everyone likes someone
vs.
There is someone who is liked by everyone

Formal Proofs

Inference Proofs

- New way of doing proofs:
  - Write down all the facts we know (givens)
  - Combine the things we know to derive new facts
  - Continue until what we want to show is a fact

- Modus Ponens
  - $[(p \rightarrow q) \land p] \rightarrow q \equiv T$
  - If you have an implication and its hypothesis as facts, you can get the conclusion

- Direct Proof Rule
  - Assume x and then eventually get y, you can conclude that $x \rightarrow y$

Inference Proof Example

Given $((p \rightarrow q) \land (q \rightarrow r))$, show that $(p \rightarrow r)$

1.	$((p \rightarrow q) \land (q \rightarrow r))$	Given
2.	$p \rightarrow q$	Eliminate $\land$: 1
3.	$q \rightarrow r$	Eliminate $\land$: 1
4.1	p	Assumption
4.2	q	Modus Ponens: 4.1, 2
4.3	r	Modus Ponens: 4.2, 3
5.	$p \rightarrow r$	Direct Proof Rule

Problem 5 - Formal Proof (Direct Proof Rule)

Show that $\neg t \rightarrow s$ follows from $t \vee q$, $q \rightarrow r$ and $r \rightarrow s$

Work on this problem with the people around you, and then we'll go over it together!

Problem 5 - Formal Proof (Direct Proof Rule)

Show that $\neg t \rightarrow s$ follows from $t \vee q$, $q \rightarrow r$ and $r \rightarrow s$

1.	$t \vee q$	Given
2.	$q \rightarrow r$	Given
3.	$r \rightarrow s$	Given
?.	$\neg t \rightarrow s$	???

Problem 5 - Formal Proof (Direct Proof Rule)

Show that $\neg t \rightarrow s$ follows from $t \vee q$, $q \rightarrow r$ and $r \rightarrow s$

1.	$t \vee q$	Given
2.	$q \rightarrow r$	Given
3.	$r \rightarrow s$	Given
4.1	$\neg t$	Assumption
?.	$\neg t \rightarrow s$	???

Problem 5 - Formal Proof (Direct Proof Rule)

Show that $\neg t \rightarrow s$ follows from $t \vee q$, $q \rightarrow r$ and $r \rightarrow s$

1.	$t \vee q$		Given
2.	$q \rightarrow r$		Given
3.	$r \rightarrow s$		Given
	4.1	$\neg t$	Assumption
	4.2	q	Eliminate $\vee$: 1, 4.1
?.	$\neg t \rightarrow s$		???

Problem 5 - Formal Proof (Direct Proof Rule)

Show that ¬t → s follows from t ∨ q, q → r and r → s

1.	t ∨ q		Given
2.	q → r		Given
3.	r → s		Given
4.1		¬t	Assumption
4.2		q	Eliminate ∨: 1, 4.1
4.3		r	Modus Ponens: 4.2, 2
?.	¬t → s		???

Problem 5 - Formal Proof (Direct Proof Rule)

Show that ¬t → s follows from t ∨ q, q → r and r → s

1.	t ∨ q		Given
2.	q → r		Given
3.	r → s		Given
	4.1	¬t	Assumption
	4.2	q	Eliminate ∨: 1, 4.1
	4.3	r	Modus Ponens: 4.2, 2
	4.4	s	Modus Ponens: 4.3, 3
?.	¬t → s		???

Problem 5 - Formal Proof (Direct Proof Rule)

Show that $\neg t \rightarrow s$ follows from $t \vee q$, $q \rightarrow r$ and $r \rightarrow s$

1.	$t \vee q$		Given
2.	$q \rightarrow r$		Given
3.	$r \rightarrow s$		Given
	4.1	$\neg t$	Assumption
	4.2	q	Eliminate $\vee$: 1, 4.1
	4.3	r	Modus Ponens: 4.2, 2
	4.4	s	Modus Ponens: 4.3, 3
5.	$\neg t \rightarrow s$		Direct Proof Rule

Problem 5 - Formal Proof (Direct Proof Rule)

```
\begin{align*}
  &\text{1. } p \vee q &&\text{given} \\
  &\text{2. } q \rightarrow r &&\text{given} \\
  &\text{3. } r \rightarrow s &&\text{given} \\
  &\hspace{5mm} \text{4.1 } \neq p &&\text{assume} \\
  &\hspace{5mm} \text{4.2 } q &&\text{Elim }\vee: 1, 4.1 \\
  &\hspace{5mm} \text{4.3 } r &&\text{MP: 2, 4.2} \\
  &\hspace{5mm} \text{4.4 } s &&\text{MP: 4.3, 3} \\
  &\text{4. } \neg p \rightarrow s &&\text{DPR}
\end{align*}
```

1. $p \vee q$	given
2. $q \rightarrow r$	given
3. $r \rightarrow s$	given
4.1 $\neg p$	assume
4.2 q	Elim $\vee : 1, 4.1$
4.3 r	MP: 2, 4.2
4.4 s	MP: 4.3, 3
4. $\neg p \rightarrow s$	DPR

Problem 6 - Find the Bug

Each of these inference proofs is incorrect. Identify the line (or lines) which incorrectly apply a law, and explain the error. Then, if the claim is false, give concrete examples of propositions to show it is false. If it is true, write a correct proof.

a) This proof claims to show that given $a \rightarrow (b \lor c)$, we can conclude $a \rightarrow c$.

1.	$a \rightarrow (b \lor c)$	Given
2.1	a	Assumption
2.2	$\neg b$	Assumption
2.3	$b \lor c$	Modus Ponens: 1, 2.1
2.4	c	Eliminate $\lor$: 2.2, 2.3
3.	$a \rightarrow c$	Direct Proof Rule

Problem 6 - Find the Bug (a)

1.	$a \to (b \lor c)$	Given
2.1	a	Assumption
2.2	$\neg b$	Assumption
2.3	$b \lor c$	Modus Ponens: 1, 2.1
2.3	c	Eliminate $\lor$: 2.2, 2.3
3.	$a \to c$	Direct Proof Rule

The error here is in lines 2.2 and 2. When beginning a subproof for the direct proof rule, only one assumption may be introduced. If the author of this proof wanted to assume both a and $\neg$ b, they should have used the assumption $a \land \neg b$, which would make line 3 be $(a \land \neg b) \to c$ instead.

And the claim is false in general. Consider:
a: "I am outside"
b: "I am walking my dog"
c: "I am swimming"
If we assert "If I am outside, I am walking my dog or swimming," we cannot reasonably conclude that "If I am outside, I am swimming" ($a \to c$).

Problem 6 - Find the Bug

Each of these inference proofs is incorrect. Identify the line (or lines) which incorrectly apply a law, and explain the error. Then, if the claim is false, give concrete examples of propositions to show it is false. If it is true, write a correct proof.

b) This proof claims to show that given $p \rightarrow q$ and r, we can conclude $p \rightarrow (q \vee r)$.

1.	$p \rightarrow q$	Given
2.	r	Given
3.	$p \rightarrow (q \vee r)$	Intro $\vee$: 1, 2

Problem 6 - Find the Bug (b)

1.	$p \to q$	Given
2.	r	Given
3.	$p \to (q \vee r)$	Intro $\vee$: 1, 2

Bug is in step 3, we're applying the rule to only a subexpression.

The statement is true though. A correct proof introduces p as an assumption, uses MP to get q, introduces $\vee$ to get $q \vee r$, and the direct proof rule to complete the argument

Problem 6 - Find the Bug

Each of these inference proofs is incorrect. Identify the line (or lines) which incorrectly apply a law, and explain the error. Then, if the claim is false, give concrete examples of propositions to show it is false. If it is true, write a correct proof.

c) This proof claims to show that given $p \rightarrow q$ and q that we can conclude p.

1.	$p \rightarrow q$	Given
2.	q	Given
3.	$\neg p \lor q$	Law of Implication: 1
4.	p	Eliminate $\lor$: 2, 3

Problem 6 - Find the Bug (c)

1.	$p \to q$	Given
2.	q	Given
3.	$\neg p \vee q$	Law of Implication: 1
4.	p	Eliminate $\vee$: 2, 3

The bug is in step 4. Eliminate $\vee$ from 3 would let us conclude $\neg p$ if we had $\neg q$ or q if we had p. It doesn't tell us anything with knowing q.

Indeed, the claim is false. We could have
p: "it is raining"
q: "I have my umbrella"
and be a person who always carries their umbrella with them (even on sunny days).
The information is not sufficient to conclude p

That’s All, Folks!

Any questions?