

Nested Quantifiers

Let our domain of discourse be $\{A, B, C, D, E\}$

And our proposition $P(x, y)$ be given by the table.

What should we look for in the table?

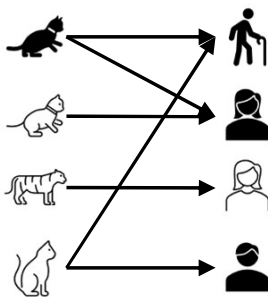
$\exists x \forall y P(x, y)$

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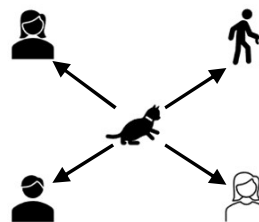
	y				
$P(x, y)$	A	B	C	D	E
A	T	T	T	T	T
B	T	F	F	T	F
C	F	T	F	F	F
D	F	F	F	F	T
E	F	F	F	T	F

Try it yourselves

Every cat loves some human.



There is a cat that loves every human.



Let your domain of discourse be mammals.

Use the predicates $\text{Cat}(x)$, $\text{Dog}(x)$, and $\text{Loves}(x, y)$ to mean x loves y .

Our First Direct Proof

Definitions

$$\text{Even}(x) := \exists k(x = 2k)$$

Prove: "For all integers x , if x is even, then x^2 is even." $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$

Direct Proof Steps

These are the usual steps. We'll see different outlines in the future!!

- Introduction
 - Declare an arbitrary variable for each \forall quantifier
 - Assume the left side of the implication
- Core of the proof
 - Unroll the predicate definitions
 - Manipulate towards the goal (using creativity, algebra, etc.)
 - Reroll definitions into the right side of the implication
- Conclude that you have proved the claim