CSE 311 Midterm Review!

Midterm Review

Administrivia

Announcements & Reminders

TONIGHT @ 6-7:30 pm in BAG 131 and 154

- Please **bring an ID** (Husky Card or other ID) to the exam. We'll be checking those during the exam.
- Remember you're allowed one piece of paper of handwritten notes. Please read details on the exams page.

- **Check your email** for room assignments

Set Theory

Problem 4- Section 04

Write an English proof, proving the following set identity

Let the universal set be U . Prove $A \cap B' \subseteq A \setminus B$ for any sets A, B.

Work on this problem with the people around you.

(1) Translate the claim to predicate
(2) Write out the skeleton

Problem 4- Section 04

Write an English proof, proving the following set identity

Let the universal set be $\mathcal{U}$. Prove $A \cap B' \subseteq A \setminus B$ for any sets A, B.

Work on this problem with the people around you.

(1) Translate the claim to predicate
$\forall x(x \in A \cap B' \rightarrow x \in A \setminus B)$

(2) Write out the skeleton

Remember the Skeleton!

How would we show $A \subseteq B$?

$$A \subseteq B \equiv \forall x(x \in A \rightarrow x \in B)$$

Let x be an arbitrary element of A

...

So x is also in B.

Since x was an arbitrary element of A, we have that $A \subseteq B$.

Let's Write it Out:

$\forall x(x \in A \cap B' \rightarrow x \in A \setminus B)$

Problem 4- Section 04

Write an English proof, proving the following set identity

Let the universal set be U . Prove A ∩ B’ ⊆ A\B for any sets A, B.

Let x be an arbitrary element and suppose that x ∈ A ∩ B’.

...

...

…So x ∈ A\B.

Since x was arbitrary, we can conclude that A ∩ B’ ⊆ A\B by definition of subset

Problem 4- Section 04

Write an English proof, proving the following set identity

Let the universal set be U . Prove A ∩ B' ⊆ A\B for any sets A, B.

Let x be an **arbitrary element** and **suppose** that x ∈ A ∩ B'.
By definition of intersection, x ∈ A and x ∈ B'
...

…So x ∈ A\B.

Since x was arbitrary, we can conclude that A ∩ B' ⊆ A\B by definition of subset

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Let the universal set be U . Prove A ∩ B' ⊆ A\B for any sets A, B.

Let x be an **arbitrary element** and **suppose** that x ∈ A ∩ B'.
By definition of intersection, x ∈ A and x ∈ B'
So by definition of complement, x ∉ B

…So x ∈ A\B.

Since x was arbitrary, we can conclude that A ∩ B' ⊆ A\B by definition of subset

Problem 4- Section 04

Write an English proof, proving the following set identity

Let the universal set be U. Prove $A \cap B' \subseteq A \backslash B$ for any sets A, B.

Let x be an **arbitrary element** and **suppose** that $x \in A \cap B'$.
By definition of intersection, $x \in A$ and $x \in B'$
So by definition of complement, $x \notin B$

Then, by definition of set difference, $x \in A \backslash B$.

Since x was arbitrary, we can conclude that $A \cap B' \subseteq A \backslash B$ by definition of subset

Strong Induction

Problem 6 - Section 06 (23sp)

Consider the function $a(n)$ defined for $n \geqslant 1$ recursively as follows.

$$a(1) = 1$$

$$a(2) = 3$$

$$a(n) = 2a(n-1) - a(n-2) \text{ for } n \geqslant 3$$

Use strong induction to prove that $a(n) = 2n - 1$ for all $n \geqslant 1$.

Work on this problem with the people around you.

Remember our Strong Induction Template!

Let $P(n)$ be "(whatever you're trying to prove)".
We show $P(n)$ holds for all $n \geq b_{min}$ by induction on n.

Base Case: Show $P(b_{min}), P(b_{min+1}), \ldots, P(b_{max})$ are all true.

Inductive Hypothesis: Suppose $P(b_{min}) \wedge \cdots \wedge P(k)$ hold for an arbitrary $k \geq b_{max}$.

Inductive Step: Show $P(k+1)$ (i.e. get $P(b_{min}) \wedge \cdots \wedge P(k) \rightarrow P(k+1)$)

Conclusion: Therefore, $P(n)$ holds for all $n \geq b_{min}$ by the principle of induction.

Let's Write it Out:

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Problem 6

Consider the function $a(n)$ defined for $n \geqslant 1$ recursively as follows.

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Use strong induction to prove that $a(n) = 2n - 1$ for all $n \geqslant 1$.

Let P(n) be "".

We show P(n) holds…

Base Cases:

Inductive Hypothesis:

Inductive Step:

Conclusion: Therefore, P(n) holds for all … by the principle of induction

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$$a(n) = 2a(n-1) - a(n-2) \text{ for } n \geqslant 3$$

Use strong induction to prove that $a(n) = 2n - 1$ for all $n \geqslant 1$.

Let P(n) be "a(n) = 2n - 1".

We show P(n) holds…

Base Cases:

Inductive Hypothesis:

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Conclusion: Therefore, P(n) holds for all … by the principle of induction

Consider the function $a(n)$ defined for $n \geqslant 1$ recursively as follows.

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Let P(n) be "a(n) = 2n - 1".

We show P(n) holds for all n ≥ 1 by induction on n.

Base Cases:

Inductive Hypothesis:

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Let P(n) be "a(n) = 2n - 1".

We show P(n) holds for all n ≥ 1 by induction on n.

Base Cases: (n = 1, n = 2) a(1) = 1 = 2(1) - 1 and a(2) = 3 = 2(2) - 1 by definition of a.

Inductive Hypothesis:

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Inductive Hypothesis: Suppose P(1) ∧ P(2) ∧ … ∧ P(k) hold for an arbitrary k ≥ 2.

i.e. a(k) = 2k - 1, a(k - 1) = 2(k - 1) - 1, a(k - 2) = 2(k - 2) - 1, etc.

Inductive Step:

Conclusion: Therefore, P(n) holds for all n ≥ 1 by the principle of induction

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Inductive Step: Goal: Show P(k + 1): a(k + 1) = 2(k + 1) - 1

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a(k + 1) = …

…

= 2(k + 1) - 1

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i.e. a(k) = 2k - 1, a(k - 1) = 2(k - 1) - 1, a(k - 2) = 2(k - 2) - 1, etc.

Inductive Step: Goal: Show P(k + 1): a(k + 1) = 2(k + 1) - 1

a(k + 1) = 2a(k) - a(k - 1) [Definition of a]

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= 2(k + 1) - 1

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Inductive Step: Goal: Show P(k + 1): a(k + 1) = 2(k + 1) - 1

a(k + 1) = 2a(k) - a(k - 1) [Definition of a]

= 2(2k - 1) - (2(k - 1) - 1) [Inductive Hypothesis]

…

= 2(k + 1) - 1

Conclusion: Therefore, P(n) holds for all n ≥ 1 by the principle of induction

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Use strong induction to prove that $a(n) = 2n - 1$ for all $n \geqslant 1$.

Problem 6

Let P(n) be "a(n) = 2n - 1".

We show P(n) holds for all n ≥ 1 by induction on n.

Base Cases: (n = 1, n = 2) a(1) = 1 = 2(1) - 1 and a(2) = 3 = 2(2) - 1 by definition of a.

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Inductive Step: Goal: Show P(k + 1): a(k + 1) = 2(k + 1) - 1

a(k + 1) = 2a(k) - a(k - 1)	[Definition of a]
= 2(2k - 1) - (2(k - 1) - 1)	[Inductive Hypothesis]
= 2k + 1	[Algebra]
= 2(k + 1) - 1	[Algebra

Conclusion: Therefore, P(n) holds for all n ≥ 1 by the principle of induction

Questions?

Topics:

- Translations & Predicate Logic
- English Proofs
- Number Theory
- Set Theory
- Strong Induction
- Weak Induction

That’s All Folks!

Breathe, you are going to do great!