CSE 311: Foundations of Computing I

Boolean Algebra Axioms (and Some Theorems)

Axioms

Closure

$a + b$ is in $\mathbb{B}$

$a \bullet b$ is in $\mathbb{B}$

Commutativity

$$a + b = b + a$$
$$a \bullet b = b \bullet a$$

Associativity

$$a + (b + c) = (a + b) + c$$
$$a \bullet (b \bullet c) = (a \bullet b) \bullet c$$

Identity

$$a + 0 = a$$
$$a \bullet 1 = a$$

Distributivity

$$a + (b \bullet c) = (a + b) \bullet (a + c)$$
$$a \bullet (b + c) = (a \bullet b) + (a \bullet c)$$

Complementarity

$$a + a' = 1$$
$$a \bullet a' = 0$$

Theorems

Null

$$X + 1 = 1$$
$$X \bullet 0 = 0$$

Idempotency

$$X + X = X$$
$$X \bullet X = X$$

Involution

$$(X')' = X$$

Uniting

$$X \bullet Y + X \bullet Y' = X$$
$$(X + Y) \bullet (X + Y') = X$$

Absorption

$$X + X \bullet Y = X$$
$$(X + Y') \bullet Y = X \bullet Y$$
$$X \bullet (X + Y) = X$$
$$(X \bullet Y') + Y = X + Y$$

DeMorgan

$$(X + Y + \cdots)' = X' \bullet Y' \bullet \cdots$$
$$(X \bullet Y \bullet \cdots)' = X' + Y' + \cdots$$

Consensus

$$(X \bullet Y) + (Y \bullet Z) + (X' \bullet Z) = X \bullet Y + X' \bullet Z$$
$$(X + Y) \bullet (Y + Z) \bullet (X' + Z) = (X + Y) \bullet (X' + Z)$$

Factoring

$$(X + Y) \bullet (X' + Z) = X \bullet Z + X' \bullet Y$$
$$X \bullet Y + X' \bullet Z = (X + Z) \bullet (X' + Y)$$