Another Proof

Claim: $\forall a(\texttt{Even}(a^2) \rightarrow \texttt{Even}(a))$ "if a^2 is even, then a is even."

See how far you get (this is somewhat a trick question).

At the very least, introduce variables, assume anything you can at the start, put down your "target" at the bottom of the paper.

Divides

Divides

For integers x, y we say $x|y$ ("x divides y") iff there is an integer z such that $xz = y$.

Which of these are true?

$2\|4$	$4\|2$	$2\| -2$
$5\|0$	$0\|5$	$1\|5$

Unique

The Division Theorem

For every $a \in \mathbb{Z}$, $\boldsymbol{d} \in \mathbb{Z}$ with $\boldsymbol{d} > \mathbf{0}$
There exist ***unique*** integers q, r with $0 \leq r < d$
Such that $a = dq + r$

"unique" means "only one"....but be careful with how this word is used.

r is unique, given a, d. – it still depends on a, d but once you've chosen a and d

"unique" is not saying $\exists r \forall a, d \quad P(a, d, r)$
It's saying $\forall a, d \exists r [P(a, d, r) \wedge [P(a, d, x) \rightarrow x = r]]$

Another Proof

For all integers, a, b, c: Show that if $a \nmid (bc)$ then $a \nmid b$ or $a \nmid c$.

Proof:

Let a, b, c be arbitrary integers, and suppose $a \nmid (bc)$.

Then there is not an integer z such that $az = bc$

…

So $a \nmid b$ or $a \nmid c$