## More proofs

Show that if $a \equiv b(\bmod n)$ and $c \equiv d(\bmod n)$ then $a c \equiv b d(\bmod n)$.

Step 1: What do the words mean?
Step 2: What does the statement as a whole say?
Step 3: Where do we start?
Step 4: What's our target?
Step 5: Now prove it.

## Another Proof

Show that if $a \equiv b(\bmod n)$ and $c \equiv d(\bmod n)$ then $a c \equiv b d(\bmod n)$.
Let $a, b, c, d, n$ be integers, $n \geq 0$
and suppose $a \equiv b(\bmod n)$ and $c \equiv d(\bmod n)$.
$n \mid(b-a)$ and $n \mid(d-c)$ by definition of mod.
$n k=(b-a)$ and $n j=(d-c)$ for integers $j, k$ by definition of divides.

$$
\begin{aligned}
& n ? ?=b d-a c \\
& n \mid(b d-a c) \\
& a c \equiv b d(\bmod n)
\end{aligned}
$$

## GCD and LCM

## Greatest Common Divisor

The Greatest Common Divisor of $a$ and $b(\operatorname{gcd}(a, b))$ is the largest integer $\boldsymbol{c}$ such that $\boldsymbol{c} \mid \boldsymbol{a}$ and $\boldsymbol{c} \mid \boldsymbol{b}$

## Least Common Multiple

The Least Common Multiple of $a$ and $b(\operatorname{lcm}(a, b))$ is the smallest positive integer $\boldsymbol{c}$ such that $\boldsymbol{a} \mid \boldsymbol{c}$ and $\boldsymbol{b} \mid \boldsymbol{c}$.

```
Try a few values...
gcd}(100,125
gcd(17,49)
gcd(17,34)
gcd(13,0)
lcm(7,11)
Icm(6,10)
```

