Let's Try Another Induction Proof

Let $g(n) = \begin{cases} 2 & \text{if } n = 2 \\ g(n-1)^2 + 3g(n-1) & \text{if } n > 2 \end{cases}$

Prove $g(n)$ is even for all $n \geq 2$ by induction on n.

Let's just set this one up, we'll leave the individual pieces as exercises.

Setup

Let $P(n)$ be "$g(n)$ is even."

We show $P(n)$ for all integers $n \geq 2$ by induction on n.

Base Case ($n = 2$): <todo>

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary $k \geq 2$.

Inductive Step:

<todo>

We conclude $P(k + 1)$. Therefore, $P(n)$ holds for all $n \geq 2$ by the principle of induction.

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Induction on Primes.

Let $P(n)$ be "n can be written as a product of primes."

We show $P(n)$ for all integers $n \geq 2$ by induction on n.

Base Case ($n = 2$): 2 is a product of just itself. Since 2 is prime, it is written as a product of primes.

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary integer $k \geq 2$.

Inductive Step:
Case 1, $k + 1$ is prime: then $k + 1$ is automatically written as a product of primes.

Case 2, $k + 1$ is composite:

Therefore $P(k + 1)$.

$P(n)$ holds for all $n \geq 2$ by the principle of induction.

Making Induction Proofs Pretty

All of our strong induction proofs will come in 5 easy(?) steps!

1. Define $P(n)$. State that your proof is by induction on n.

2. Base Case: Show $P(b)$ i.e. show the base case

3. Inductive Hypothesis: Suppose $P(b) \wedge \cdots \wedge P(k)$ for an arbitrary $k \geq b$.

4. Inductive Step: Show $P(k + 1)$ (i.e. get $[P(b) \wedge \cdots \wedge P(k)] \rightarrow P(k + 1)$)

5. Conclude by saying $P(n)$ is true for all $n \geq b$ by the principle of induction.