Proof by Contradiction Skeleton

Claim: p is true.

- Suppose for the sake of contradiction $\neg p$.
- …
- Then some statement s must hold.
- …
- And some statement $\neg s$ must hold.
- But s and $\neg s$ is a contradiction. So p must be true.

My tire is leaking

Suppose my tire is not leaking

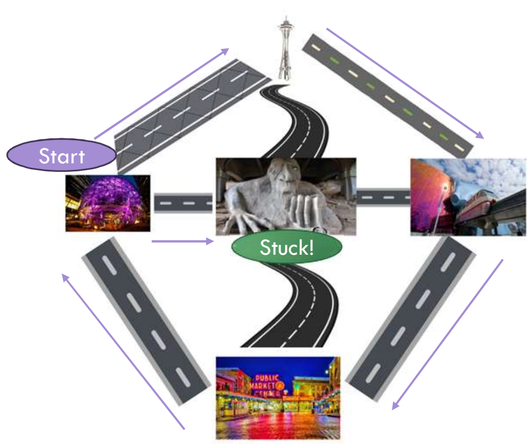
The tire pressure must be constant

The tire pressure is decreasing

My Tire is leaking

Graph Example

Can we travel on every road, without going on a road twice?



Another Proof by Contradiction

Claim: There are infinitely many primes

Proof:

Suppose for the sake of contradiction, there are only finitely many primes. Call them $p_1, p_2, \dots, p_k$.

Where can we find a contradiction?

- Show our list is non inclusive (i.e create a different prime number)
- Show one of the numbers in our list is not prime
- Create a contradiction with facts about prime factorization
- Show 1 = 2
- Show p is odd and even at the same time
- Proof by cases with a mix of the above

But [] is a contradiction! So, there must be infinitely many primes.

Another Proof by Contradiction

Claim: There are infinitely many primes

Proof:

Suppose for the sake of contradiction, there are only finitely many primes. Call them $p_1, p_2, \dots, p_k$.

Consider the number $\mathrm{q} = p_1 \cdot p_2 \cdot \dots \cdot p_k + 1$

Case 1: q is prime:

Case 2: q is not prime (i.e composite):

Since q is composite, we know that some prime p_i must divide q. This means that $q \,\%\, p_i = 0$.

Also, notice that $\mathrm{q} \,\%\, p_i = (p_1 \cdot p_2 \cdot \dots \cdot p_k) + 1 \;\%\, p_i$ using the definition of q, which gives us:

$$\mathrm{q} \,\%\, p_i = (p_1 \cdot p_2 \cdot \dots \cdot p_k) + 1 \,\%\, p_i$$

In both cases, this is a contradiction! So, there must be infinitely many primes.