CSE312, Spring 2011: Foundations of Computing II

Homework 2

Due in class on 29 April 2011

1. A salesman has two kinds of encyclopedias: the Guide ($500) and the Galactica ($1000). The salesman has two appointments. The probability of making a sale at the first appointment is 0.3, and the probability at the second appointment is 0.6 (the two sales are independent). Each sale is equally likely to be for the Guide and the Galactica (but not both). X is the total dollar value of all sales. Find the following:

   - The probability mass function of X.
   - The cumulative distribution function of X.
   - $\mathrm{E}[X]$
   - $\mathrm{Var}(X)$

   The salesman successfully made a sale in his first appointment. Given this new information, what is the conditional PMF of X?

   [4+4+4+4+4]

2. Consider a fair k-sided die.

   - What is the expected outcome of a roll?
   - You roll the die n times. What is the expected sum of the outcomes?

   (Find closed-form solutions.)

   [4+4]

3. Is it true in general that $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$? Prove or disprove the statement.

   [5]

4. An ISP uses 50 connections to serve the needs of 1000 customers. At a given time, each customer will need a connection with a probability of 0.1, independently of other customers.

   - What is the PMF of the number of connections in use?
   - The ISP claims that the probability of more than 50 people trying to connect simultaneously is less than 1 in 100 million. Is this true? Why or why not?

   [4+4]

5. You are playing a tower defence game on your laptop. On average, 15 enemy units attack your base per minute. Each unit has a 5% chance of getting through your defences (independent of other units). If three or more units get through your defences, you lose the game.

   - What is the probability that the first attack happens exactly 10 seconds into the game?
   - What is the probability that no attacks happen within 10 seconds of the start of the game?

- What is the probability that exactly 10 units attack within some specific 30 second window?
- Let X be the number of attacking units within a specific 30 second window. What are $\mathrm{E}[X]$ and $\mathrm{Var}(X)$?
- Let Y be the number of attacks before some enemy unit breaches your defences. What are $\mathrm{E}[Y]$ and $\mathrm{Var}(Y)$?
- Let Z be the number of attacks before you lose the game. What are $\mathrm{E}[Z]$ and $\mathrm{Var}(Z)$?

[4+4+4+4+4+4]

6. Consider the following functions, where λ is a positive scalar and $x \in (-\infty; \infty)$.

$$
\begin{aligned}
f(x) &= \frac{1}{2\lambda} \exp\left(-\frac{x}{\lambda}\right) \\
g(x) &= \frac{1}{2\lambda} \exp\left(-\frac{|x|}{\lambda}\right) \\
h(x) &= \lambda \exp(-\lambda x)
\end{aligned}
$$

Are each of these valid probability density functions? Why or why not?

[5+5+5]

7. A Roulette wheel is divided into 38 compartments, each of which is $360/38 = 9.47$ degrees wide. The ball lands somewhere along the rim of the wheel, according to a uniform distribution. What is the probability that the ball lands within 2 degrees of some edge between two compartments?

[4]

8. The average daily high temperature for Seattle in April is 58 °F, with a standard deviation of 4 °F. Assume the temperatures are distributed normally.

- What is the probability that the high temperature on a randomly chosen day will be greater than 65 degrees?
- What is the probability that the high temperature will be between 50 and 51 degrees?

[4+4]

9. You have a fair coin and a fair 6-sided die.

- Let X be the indicator variable for the coin landing on heads, and Y be the outcome of the die roll. What is the joint PMF of X and Y?
- Let Z be the number of heads in 2 coin flips. What is the joint PMF of Z and Y?

[4+4]