CSE312, Spring 2011: Foundations of Computing II

Homework 4

Due in class on 27 May 2011

Guidelines: You can brainstorm with others, but please solve the problems and write up the answers by yourself. You may use textbooks, lecture notes, Wikipedia etc., but please don't use any other resources or references (e.g. online problem solutions) without asking. Show your reasoning for all questions.

1. In a study of maximal aerobic capacity [*J. Applied Physiology 65,6 (December 1988), p. 2696*], the blood plasma volumes (in liters) of 12 subjects were found to be:

3.15	2.99	2.77	3.12	2.45	3.85
2.99	3.87	4.06	2.94	3.53	3.20

   (a) Using unbiased estimators, give point estimates of the mean μ and variance σ^2 of this distribution.

   (b) Assuming the data is normally distributed, give a 90% confidence interval for μ.

   [4+4]

2. Let $f(x|\theta) = \theta x^{\theta-1}$, $0 < x < 1$, $0 < \theta < \infty$. Let $X_1, \ldots, X_n$ be n i.i.d. samples from this distribution.

   (a) Derive the maximum likelihood estimator θ_{MLE}.

   (b) For the following set of samples, calculate the value of θ_{MLE}:

0.0256	0.3051	0.0278	0.8971	0.0739
0.3191	0.7379	0.3671	0.9763	0.0102

   (c) For the same set of samples, calculate the method of moments estimate θ_{MOM}.

   [4+4+4]

3. Let $X_1, \ldots, X_n$ be n i.i.d. samples from a geometric distribution with success probability p.

   (a) Find the maximum likelihood estimate of p, θ_{MLE}.

   (b) Find the method of moments estimate, θ_{MOM}.

   (c) Find the Laplace estimate, θ_L.

   (d) Find the MAP estimate, θ_{MAP}, given a Beta $(2, 10)$ prior over p.

   [5+5+5+5]

4. Let $\hat{\theta}$ be an estimate of parameter θ of some distribution.

   (a) Show that if $\mathrm{E}\left[|\theta - \hat{\theta}|^2\right] \to 0$ as $n \to \infty$, then the estimator $\hat{\theta}$ is consistent. (Hint: Use a proof by contradiction.)

   (b) Consider $X_1, \ldots, X_n \sim \mathrm{Uni}(0, \theta)$. Find the MLE $\hat{\theta}_n$ after these n samples.

   (c) Give an exact mathematical expression for the cumulative distribution function of $\hat{\theta}_n$. Also compute the probability density function.

   (d) Use this to prove that $\hat{\theta}_n$ is consistent. (Hint: Use the first part of the problem.)

[5+5+5+5]

5. Use Karatsuba's method to multiply 1001_2 by 0110_2. Carefully show the recursion tree and give a table showing (a) the problem being solved at each node, (b) the result returned for that problem, and (c) the calculations supporting that result.

   [10]

6. Suppose you are choosing between the following three algorithms:

   (a) Algorithm A solves problems by dividing them into five subproblems of half the size, recursively solving each subproblem, and then combining the solutions in linear time.

   (b) Algorithm B solves problems of size n by recursively solving two subproblems of size $n-1$ and then combining the solutions in constant time.

   (c) Algorithm C solves problems of size n by dividing them into nine subproblems of size $n/3$, recursively solving each subproblem, and then combining the solutions in $O(n^2)$ time.

   What are the running times of each of these algorithms (in big-O notation), and which would you choose?

   [15]

7. You have an array A of n elements, and you wish to remove all duplicates.

   (a) Come up with a simple $O(n^2)$ algorithm to remove duplicates from the array.

   (b) Design a more efficient divide-and-conquer algorithm to do the same thing.

   (c) Find the running time of your algorithm (in big-O notation).

   [15]