CSE 312

Foundations of Computing II

Lecture 28: Clustering (mixture models) + glimpse of auction theory

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Slide Credit: Based on Stefano Tessaro's slides for 312 19au incorporating ideas from myself ☺

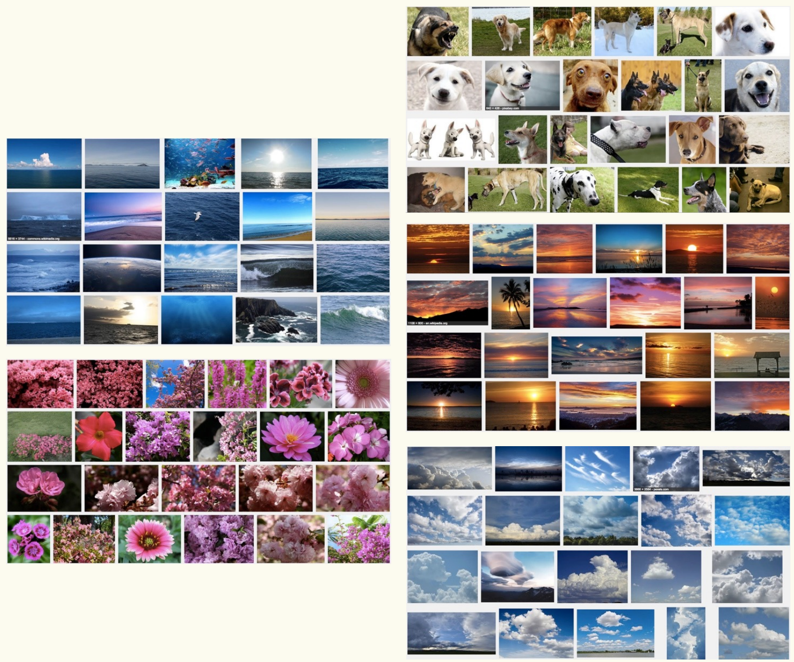
Agenda

- Mixture models and clustering
- A glimpse of auction theory

Motivating application: Clustering images

Discover groups of similar images

- Ocean
- Pink flower
- Dog
- Sunset
- Clouds
- …



Motivates probabilistic model: Mixture model

- Take uncertainty in assignment into account
  e.g., when clustering documents, might want to say 54% chance document is world news, 45% science, 1% sports, and 0% entertainment

- Allow for cluster shapes not just centers

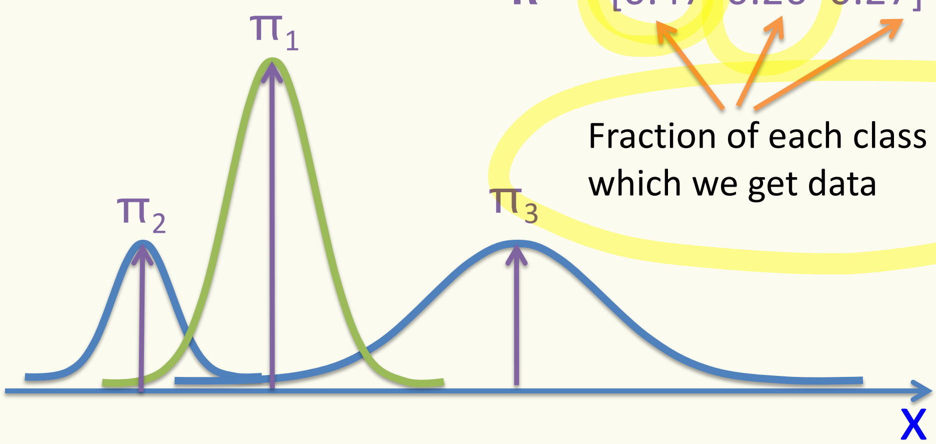
- Enables learning different weightings of dimensions
  - e.g., how much to weight each word in the vocabulary when computing cluster assignment

Combination of weighted Gaussians

Associate a weight π_k with each Gaussian component

$$\pi = [\overset{\pi_1}{0.47}\ \ \overset{\pi_2}{0.26}\ \ \overset{\pi_3}{0.27}]$$

Fraction of each class in world from which we get data



$$0 \le \pi_k \le 1$$

$$\sum_{k=1}^{K} \pi_k = 1$$

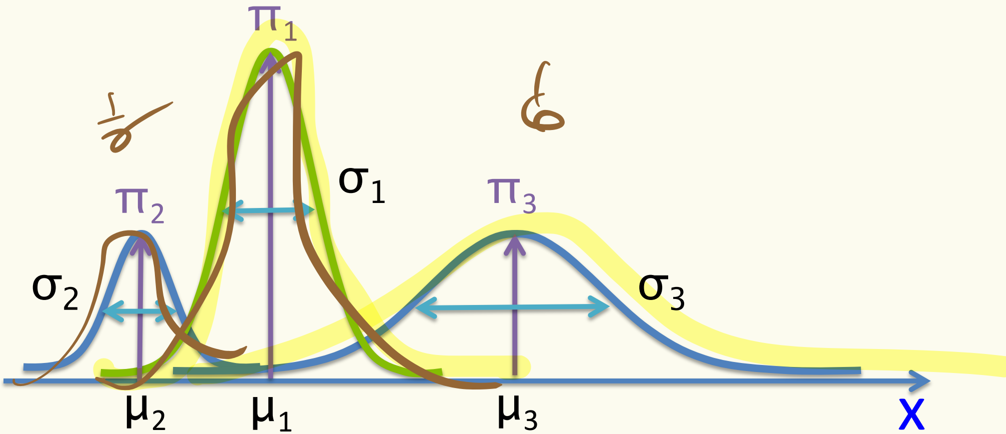
Mixture of Gaussians (1D)


[R = 0.05, G = 0.7, B = 0.9]


[R = 0.85, G = 0.05, B = 0.05]


[R = 0.02, G …]

Each mixture component represents a unique cluster specified by: $\{\pi_k, \mu_k, \sigma_k^2\}$

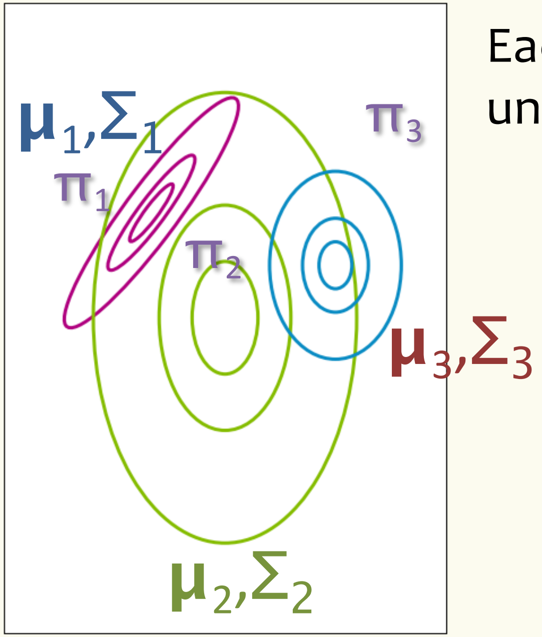


Mixture of Gaussians (general)

[R = 0.05, G = 0.7, B = 0.9]

[R = 0.85, G = 0.05, B = 0.35]

[R = 0.02, G = 0.95, B = 0.4]



Each mixture component represents a unique cluster specified by:

$$\{\pi_k, \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k\}$$

Mixture model

- K clusters, defined by the following **unknown** parameters

$$\Theta = \{\pi_j, \boldsymbol{\mu}_j, \Sigma_j\}_{j=1}^{k}$$

$$\sum_{j=1}^{k} \pi_j = 1.$$

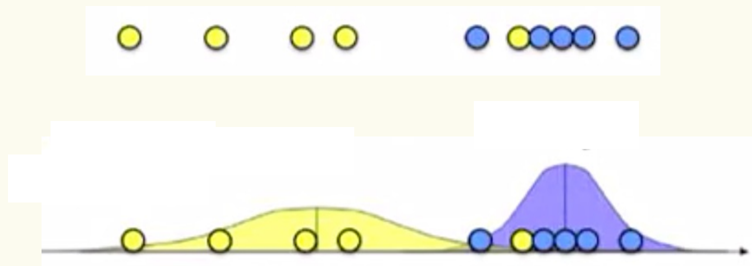
- Problem: Assume that the data comes from such a distribution, and recover the parameters of the distribution (e.g. MLE)
- Determine, for each point, the likelihood of it belonging to cluster j, for each j.

π_Y $\quad$ $\pi_B = 1-\pi_Y$

$\mu_Y, \sigma^2_Y \leftrightarrow \mu_B, \sigma^2_B$

Two1-D Gaussians, with unknown mean and variance

- Easy if know the source of each data point.

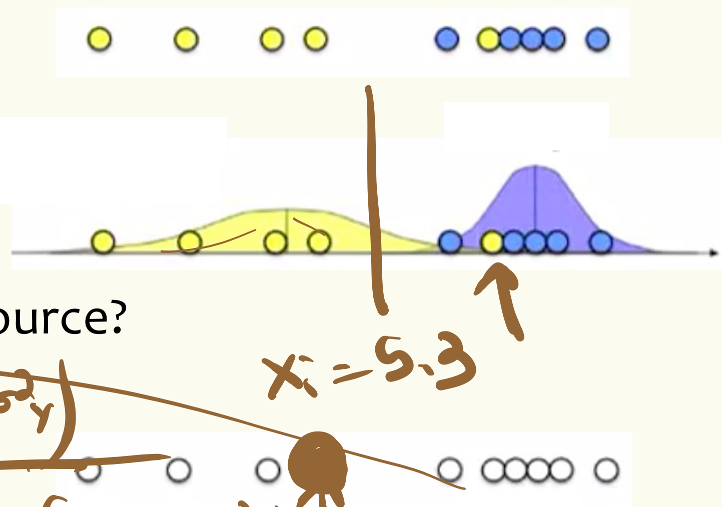


Two1-D Gaussians, with unknown mean and variance

- Easy if know the source of each data point.

π_Y

$\mu_Y \; \sigma^2_Y \quad \mu_B \; \sigma^2_B$



- What if we don't know the source?

$x_i = 5.3$

$$\text{prob}(x_i \text{ is yellow}) = \frac{f_Y(x_i; \mu_Y, \sigma^2_Y)}{f_Y(x_i; \mu_Y, \sigma^2_Y) + f_B(x_i; \mu_B, \sigma^2_B)}$$

Mixture model

- K clusters, defined by the following parameters

$$\boldsymbol{\Theta} = \{\pi_j, \boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j\}_{j=1}^{k}$$

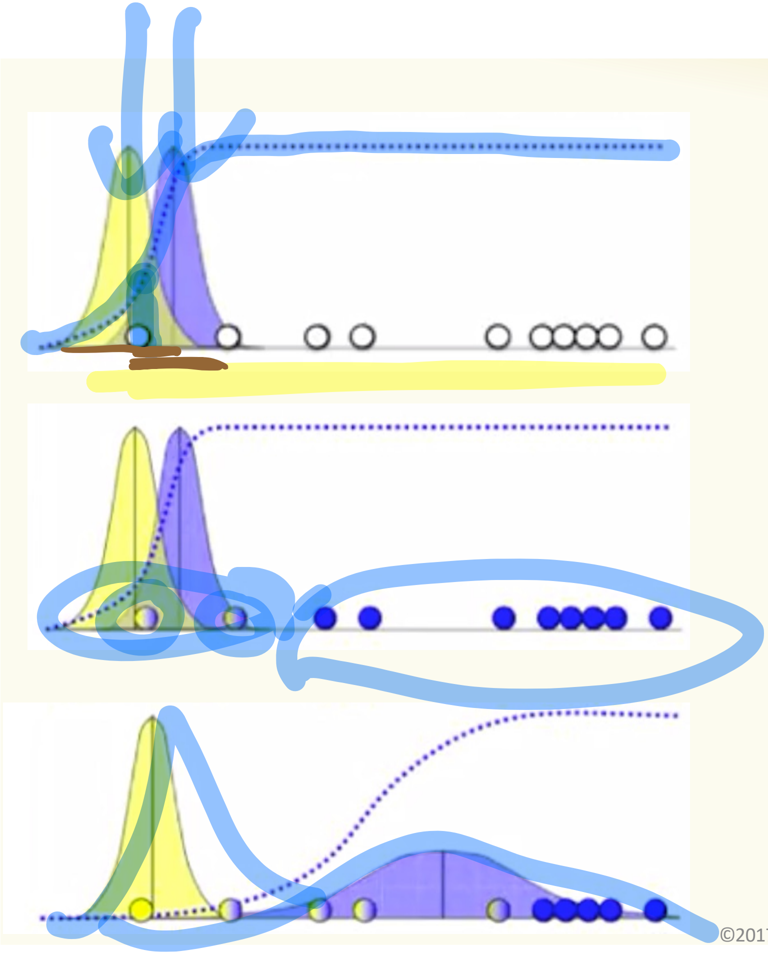
$$\sum_{j=1}^{k} \pi_j = 1.$$

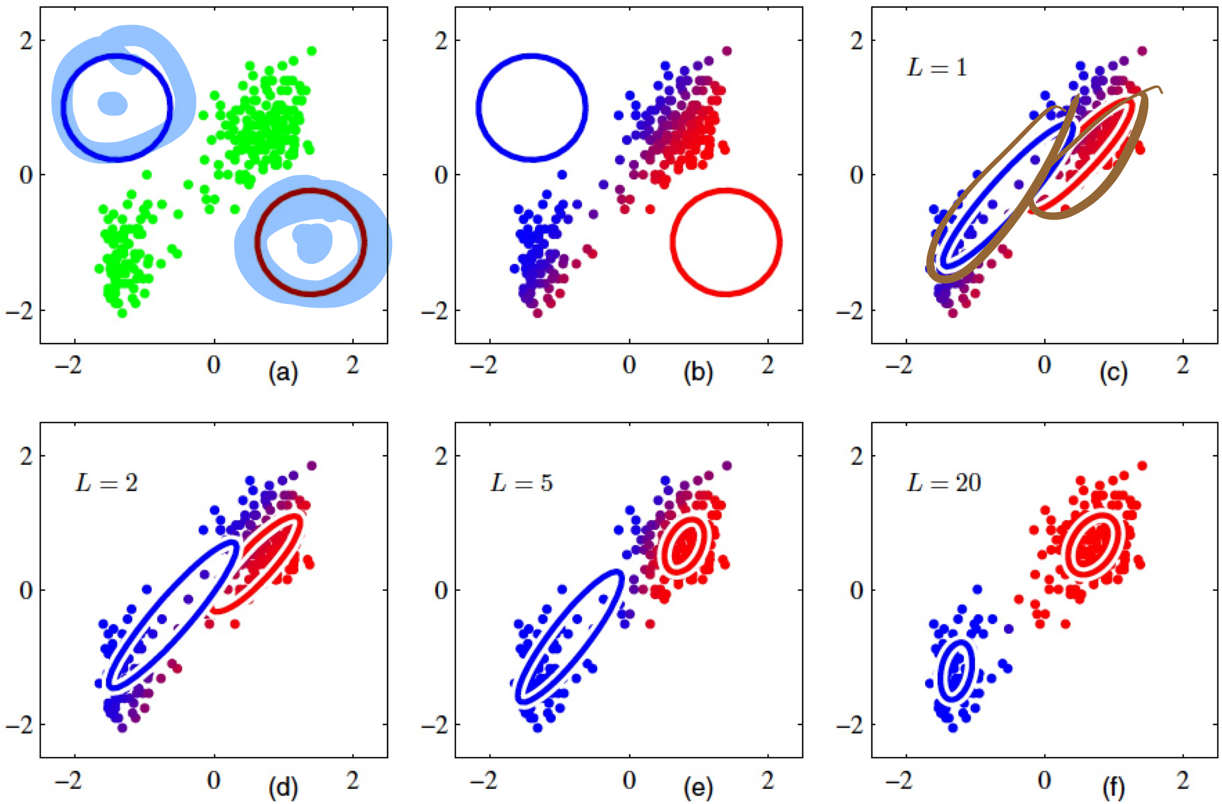
- Problem: Assume that the data comes from such a distribution, and recover the parameters of the distribution.
- Determine, for each point, the likelihood of it belonging to cluster j, for each j.
- **PROBLEM: no closed form solution**

Expectation Maximization Algorithm

Two step approach based on following observation

- If we knew which cluster each sample was from, we could estimate all the parameters.
- If we knew all the parameters we could estimate the chance each point came from each cluster.
- EM is an iterative algorithm that alternates between these two steps.





2
0
−2
−2
0
2
(a)
(b)
(c)
(d)
(e)
(f)
L = 1
L = 2
L = 5
L = 20

Agenda

- Mixture models and clustering
- A glimpse of auction theory

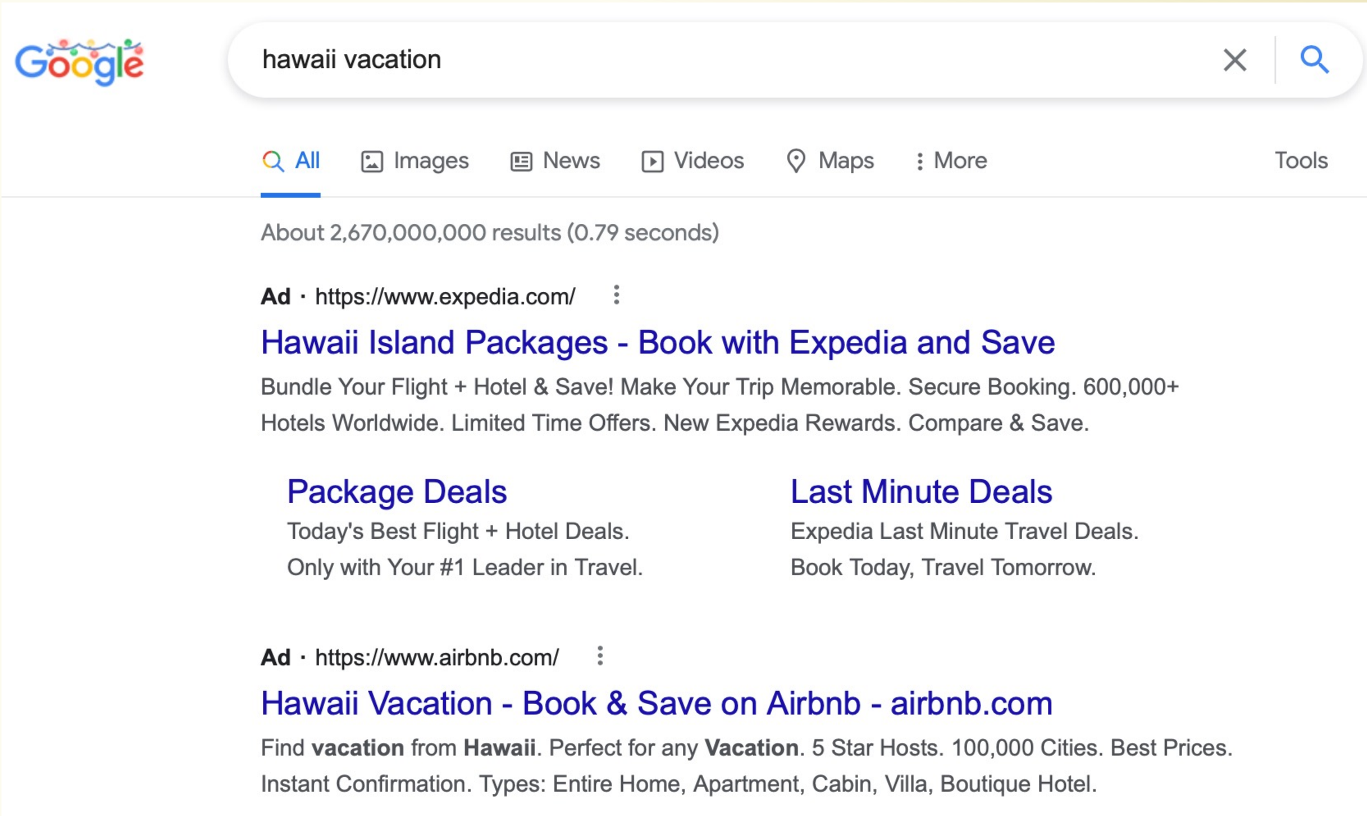
Auctions

- Some goods on eBay and amazon are sold via auction.
- Companies like Google and Facebook make most of their money by selling ads.
- The ads are sold via auction.
    - Advertisers submit bids for certain “keywords”

Facebook Ads bidding… 🤔 Is this an auction?

Yes! That’s the first thing you need to understand to master bidding management of Facebook Ads. **When you’re creating a new campaign, you’re joining a huge, worldwide auction.**

You’ll be competing with hundreds of thousands of advertisers to buy what Facebook is selling: Real estate on the News Feed, Messenger, Audience Network, and mobile apps to display your ads to the users.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Package Deals
Today's Best Flight + Hotel Deals. Only with Your #1 Leader in Travel.
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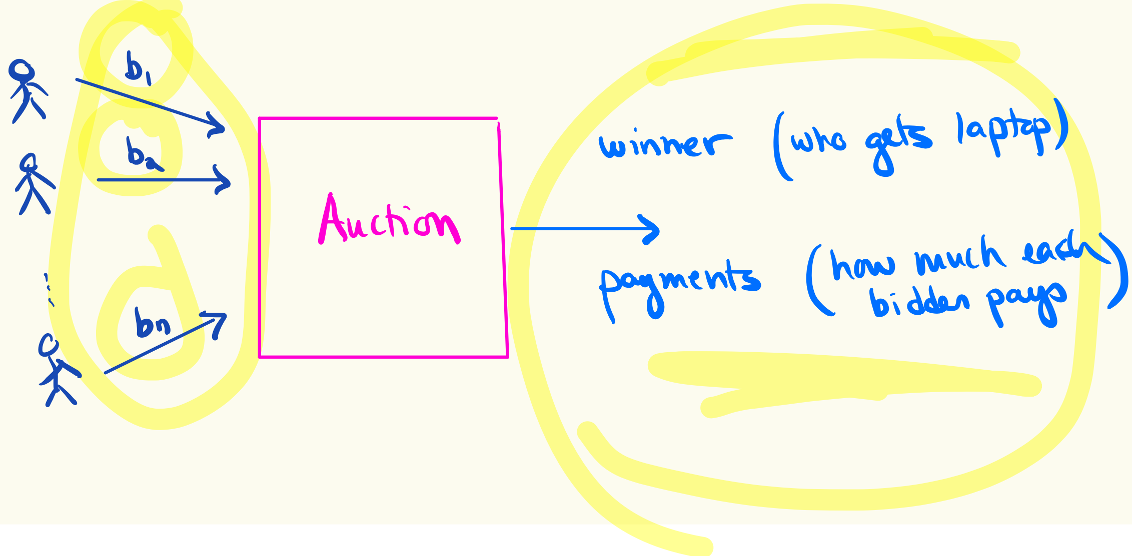
An auction is a …

- Game
  - Players: advertisers
  - Strategy choices for each player: possible bids
  - Rules of the game – made up by Google/Facebook/whoever is running the auction
- What do we expect to happen? How do we analyze mathematically?

Special case: Sealed bid single item auction

- Say I decide to run an auction to sell my laptop and I let you be the bidders.
- If I want to make as much money as possible – what should I choose as the rules of the auction?



Special case: Sealed bid single item auction

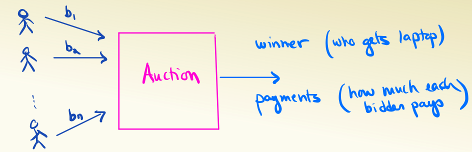
- Say I decide to run an auction to sell my laptop and I let you be the bidders.
- If I want to make as much money as possible – what should the rules of the auction be?

Some possibilities:

- **First price auction:** highest bidder wins; pays what they bid.
- **Second price auction:** highest bidder wins; pays second highest bid.
- **All pay auction:** highest bidder wins: all bidders pay what they bid.

Which of these will make me the most money?

Special case: Sealed Bid single item auction



Some possibilities:

- **First price auction:** highest bidder wins; pays what they bid.
- **Second price auction:** highest bidder wins; pays second highest bid.
- **All pay auction:** highest bidder wins: all bidders pay what they bid.

	Bidder	1	2	3	4
	Bids	100	81	35	24
Payments	1st price	100	0	0	0
	2nd price	81	0	0	0
	All pay	100	81	35	24

Bidder model

Each bidder has a value, say v_i for bidder i.

Bidder is trying to maximize their "utility" –
the value of the item they get – price they pay.

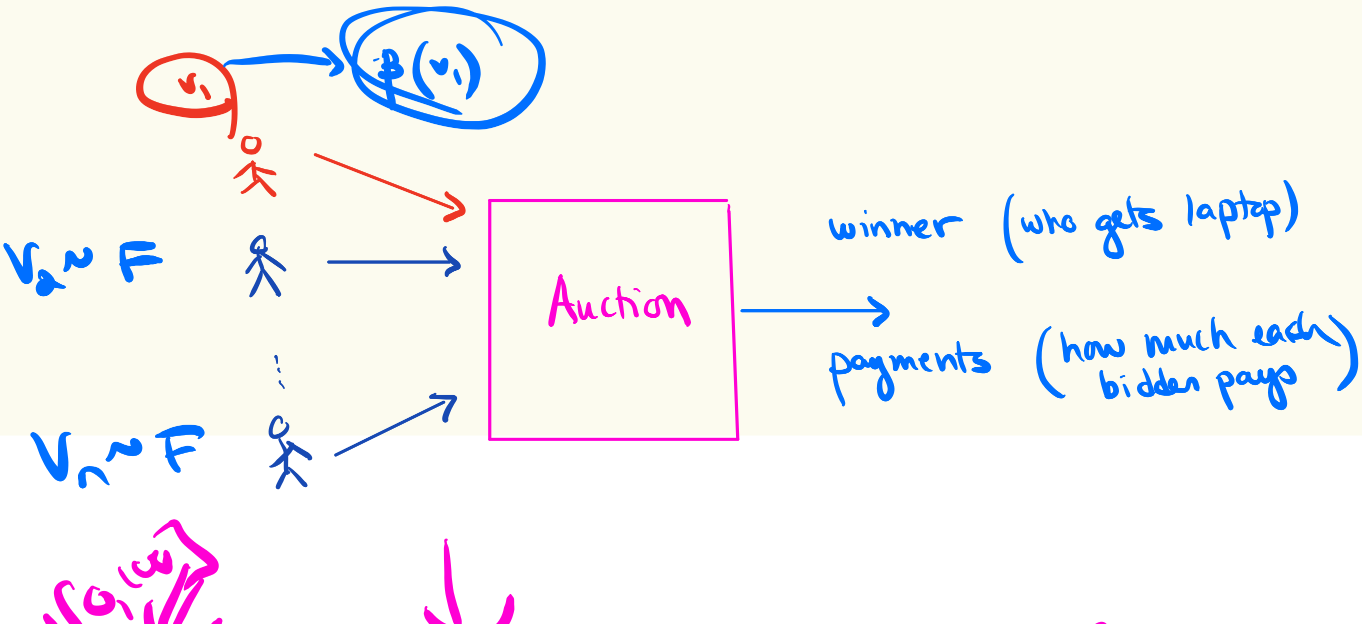
v 100 pay 99 utility = $1
100 pay 102 ut -$2

Theorem

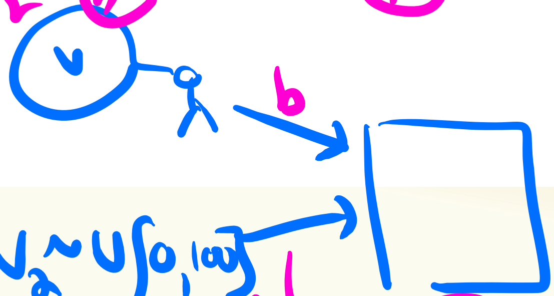
A second price auction is **truthful**. In other words, it is always in each bidder's best interest to bid their true value.

Bayes-Nash equilibrium

Suppose that $V_1 \sim F_1, V_2 \sim F_2, \ldots, V_n \sim F_n$.
A bidding strategy $\beta_i(\cdot)$ is a **Bayes-Nash equilibrium** if $\beta_i(v_i)$ is a **best response in expectation** to $\beta_j(V_j)\ \forall j \neq i$.



$V_1 \sim U$ ... first price.



$V_2 \sim U[0,100]$

$F(x) = \frac{x}{100}$

$\beta(v_2) = \frac{v_2}{2}$ $\quad \frac{v}{3}$

$E(\text{utility}) = (v-b)\Pr(\text{I win})$

$= (v-b)\Pr\left(b > \frac{v_2}{2}\right)$ r.v.

$(v-b)\Pr(v_2 < 2b)$ $\quad F(2b) = \frac{2b}{100}$

$= (v-b)\frac{b}{50}$

choose b to max my exp utility.

$b = \frac{v}{2}$ V_1

$V_1 \sim U[0,100]$

$V_2 \sim U[0,100]$

	2nd price	1st price	All pay
v	v	$\frac{v}{2}$	$\frac{v^2}{200}$
Exp auctioneer revenue:	$E[\min(V_1, V_2)]$	$E[\max(\frac{V_1}{2}, \frac{V_2}{2})]$	$E[\frac{V_1^2}{200} + \frac{V_2^2}{200}]$

Revenue Equivalence Theorem

In equilibrium, no matter what distribution the bids are drawn from, the expected auctioneer revenue is the same in all three auctions!