Probability

The Naive Bayes Classifier

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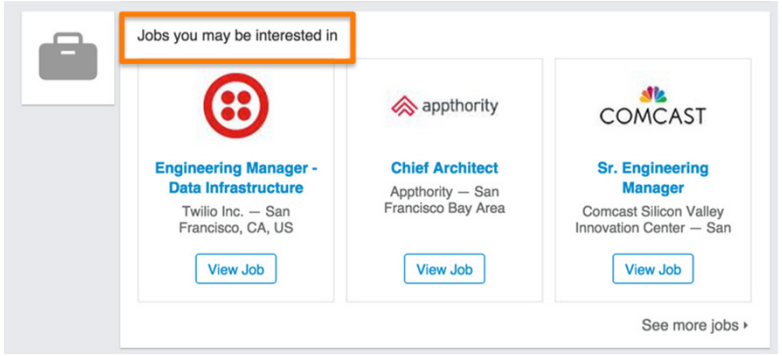
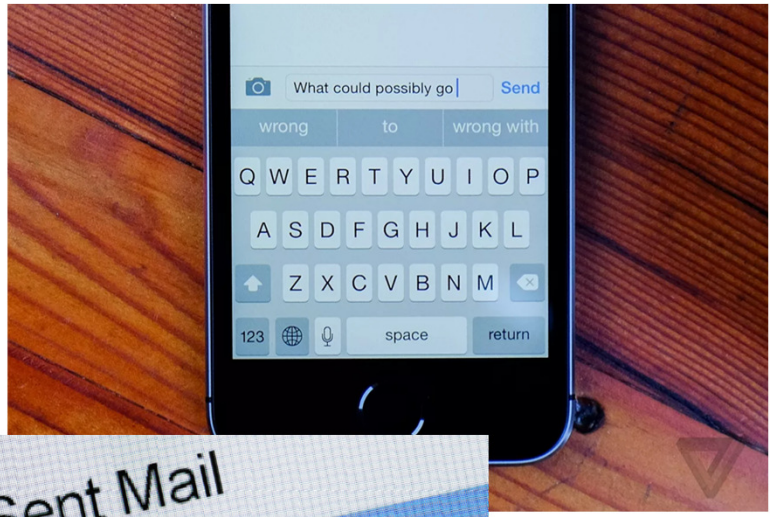
Alex Tsun

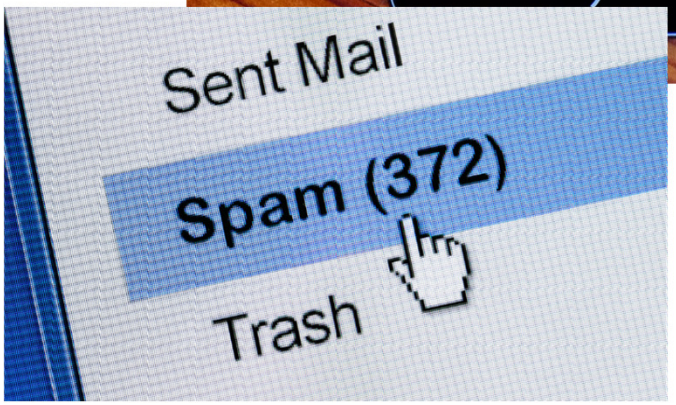
Agenda

- What is Machine Learning?
- Featurizing Emails
- Naive Bayes

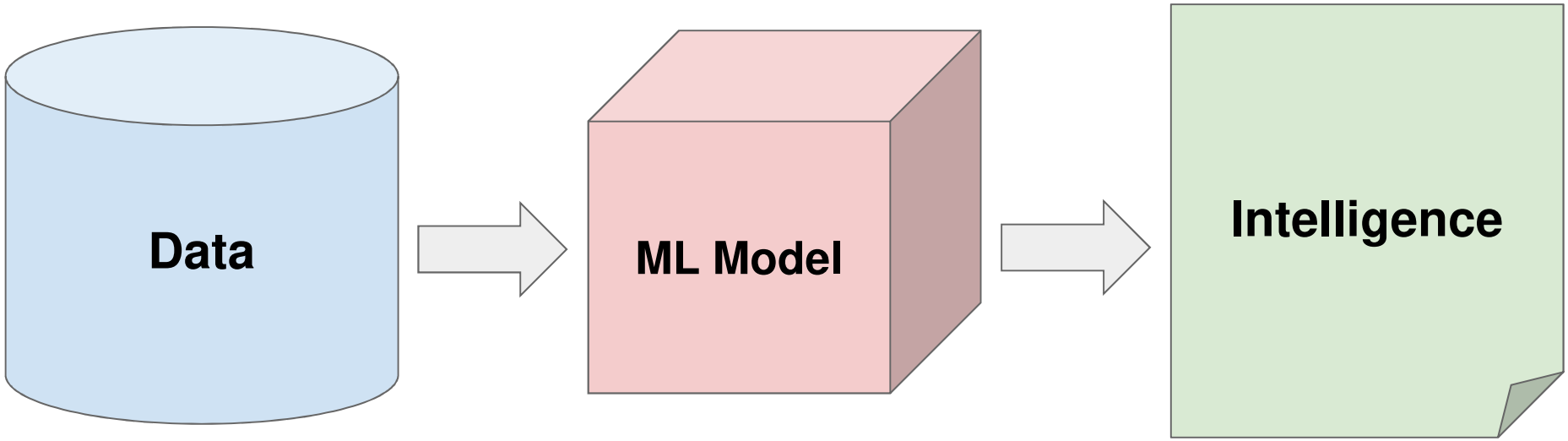
Machine Learning in the Real World







ML Pipeline



From **Wikipedia**: “Machine learning is the study of computer algorithms that improve automatically through experience.”

You are a machine!

Number	Shape	"Label"
3		12
5		15
-2		-8
7		21
-4		???

Given examples with correct "labels", make predictions!

You are a machine!

Number	Shape	"Label"
3		12
5		15
-2		-8
7		21
-4		-16

Given examples with correct "labels", make predictions!

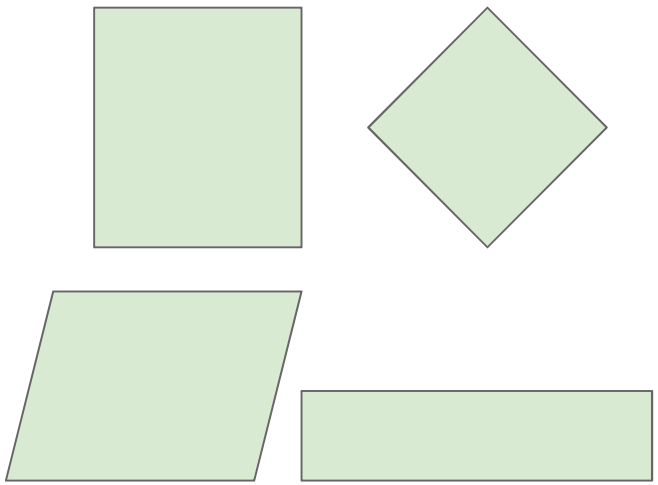
Regression: Idea

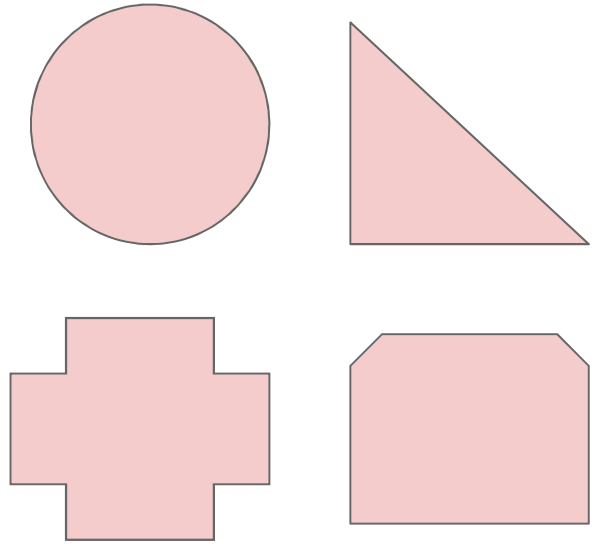
$ 340,135

$801,353

??????

Classification: Idea

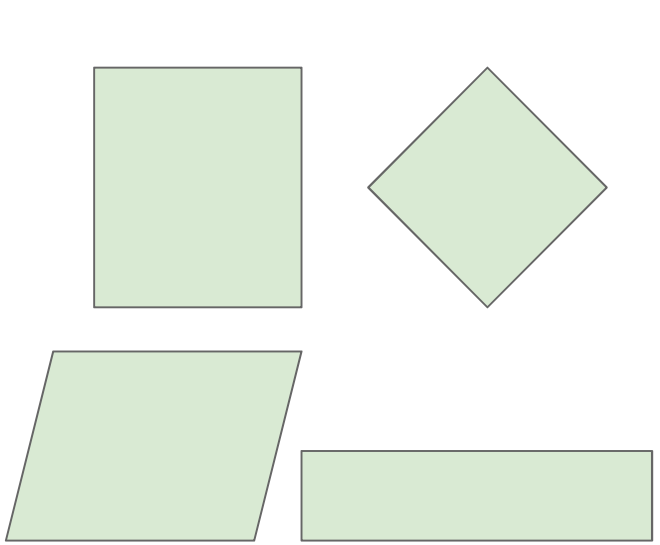


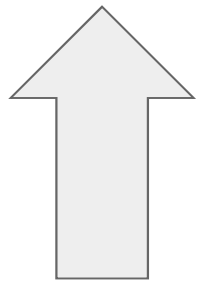


“Green” class

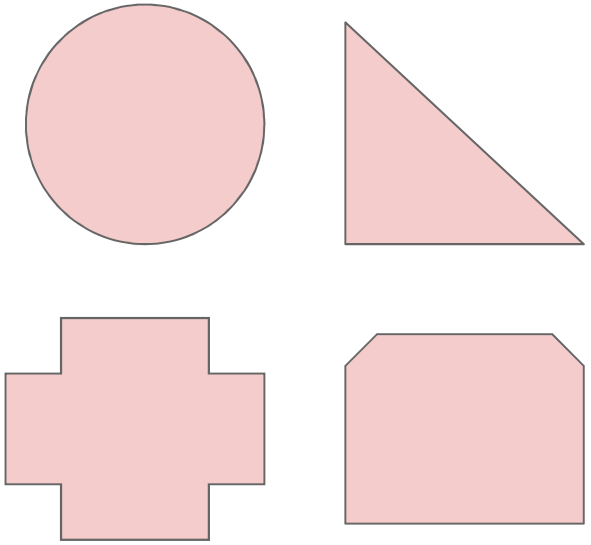
“Red” class

Classification: Idea





Is this new shape supposed to be “green” or “red”?

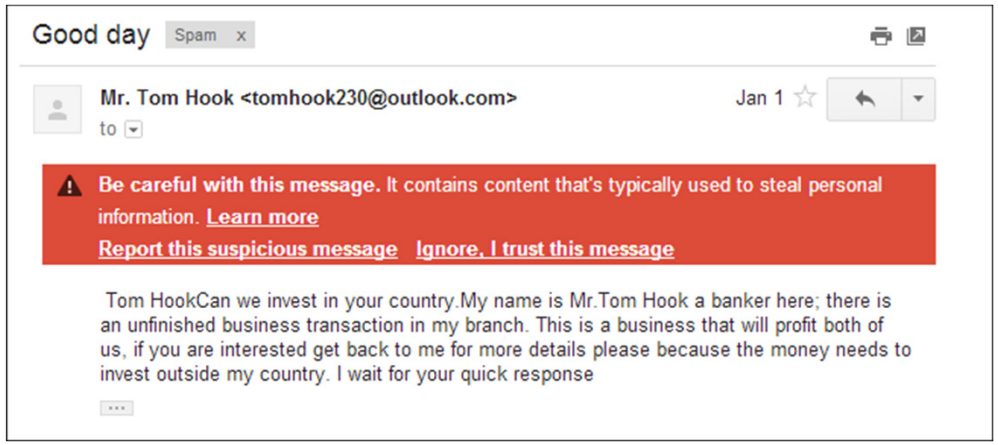


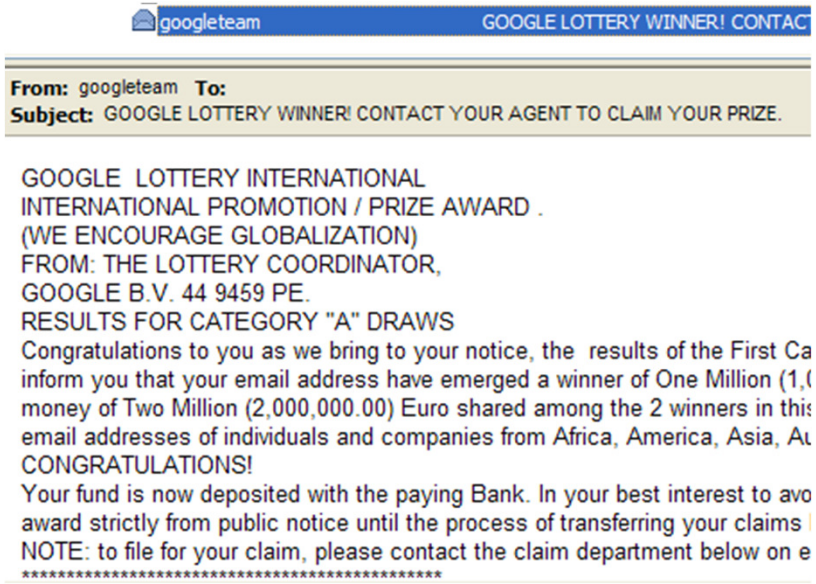
“Green” class

“Red” class

Spam Filter

- In real life, you may have seen a lot of spam emails like this.
- Building a good spam filter helps protect users from potential scams, unnecessary advertising, or malware links.





Evaluating Performance

Training Set

Email	**Label**
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

Test Set

Email	**Label**
You buy valium!	Spam
You need valium sir.	Spam
I hope you are healthy.	Ham
...	...
...	...

We "**train**" our spam filter on the training set, and **evaluate** performance using a test set (data that is unseen by the spam filter initially). This gives an unbiased estimate of performance.

Spam Filter Task

Training Set

Email	**Label**
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

Predict whether this email is spam or ham:

You buy Valium!

Emails as word collections

Email	Set of Words in the Email
SUBJECT: Top Secret Business Venture Dear Sir. First, I must solicit your confidence in this transaction, this is by virtue of its nature as being utterly confidential and top secret…	{top, secret, business, venture, dear, sir, first, I, must, solicit, your, confidence, in, this, transaction, is, by, virtue, of, its, nature, as, being, utterly, confidencial, and}

For simplicity, we will

- Ignore Duplicate Words
- Ignore Punctuation
- Ignore Casing

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Hello hello hello there.	{hello, there}

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Hello hello hello there.	{hello, there}
You buy Valium!	{you, buy, valium}

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Our approach

Compute and Compare:

$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$ $\mathbb{P}(\text{ham} \mid \text{"You buy Valium!"})$

Then predict whichever is larger! Can we get away with just computing one of them?

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Equivalently, note that these add to 1, so we can just compute

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$$

and if it is greater than 0.5, then we predict **spam**.

Otherwise, we predict **ham**.

Note: We resolve the tie in favor of **ham**.

Naive Bayes Classifier - The bayes part

Bayes Theorem: $$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(B \mid A)\,\mathbb{P}(A)}{\mathbb{P}(B)}$$

Apply it to our example:

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"}) = \frac{\mathbb{P}(\text{"You buy Valium!"} \mid \text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"You buy Valium!"})}$$

Naive Bayes Classifier - What we Calculate

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"}) = \frac{\mathbb{P}(\text{"You buy Valium!"} \mid \text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"You buy Valium!"})}$$

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$$= \frac{\mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{spam})\ \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$ [LTP]

Naive Bayes Classifier - What we Calculate

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$$\mathbb{P}(\text{spam}) = \frac{\text{total spam emails (in training set)}}{\text{total emails (in training set)}} \qquad \mathbb{P}(\text{ham}) = \frac{\text{total ham emails (in training set)}}{\text{total emails (in training set)}}$$

(our approximation for these probabilities, based on the training set)

Naive Bayes Classifier - The naive part

It is somewhat unlikely that we have the email ”You buy Valium!” in our training data. (In this case we don’t!)

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$$\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam})$$

$$\approx \mathbb{P}(\text{"you"} \mid \text{spam})\mathbb{P}(\text{"buy"} \mid \text{spam})\mathbb{P}(\text{"valium"} \mid \text{spam})$$

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Then we estimate for example that

$$\mathbb{P}(\text{“you”} \mid \text{spam}) = \frac{\text{number of spam emails containing “you” (in training set)}}{\text{number of spam emails (in training set)}}$$

Why is this Naive?

Consider for example the following two emails:

“!!!Lunch free for You!!!!!” Spam

“You free for lunch?” Ham

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One shortfalling of our model is that it will make the same prediction for these since they have the same set of words!

Example

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$$

$$= \frac{\mathbb{P}(\{\text{"you","buy","valium"}\} \mid \text{spam})\ \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you","buy","valium"}\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you","buy","valium"}\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"valium"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"valium"}|\text{spam})\mathbb{P}(\text{spam})+\mathbb{P}(\text{"you"}|\text{ham})\mathbb{P}(\text{"buy"}|\text{ham})\mathbb{P}(\text{"valium"}|\text{ham})\mathbb{P}(\text{ham})}$$

Email	**Label**
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Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

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$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = \quad \mathbb{P}(\text{"valium"} \mid \text{ham}) =$$

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= 1 (Marked as spam since no ham email contained "buy")

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What happens if we got a 0?

P("You buy Valium!" | ham) = 0 since P("buy"| ham) = 0, since no ham email in our training data contained the word '**buy**'.

But does that mean we will never encounter a ham email with word '**buy**'?

Laplace smoothing

Pretend in spam emails (training set):

- We saw one extra spam email **with** word w_i
- We saw one extra spam email **without** word w_i

Laplace smoothing

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- We saw one extra spam email **with** word w_i
- We saw one extra spam email **without** word w_i

$$\mathbb{P}(w_i \mid \text{spam}) = \frac{\left|\text{total spam emails (training set) containing } w_i\right| + 1}{\left|\text{total spam emails (training set)}\right| + 2}$$

Laplace smoothing

Pretend in spam emails (training set):

- We saw one extra spam email **with** word w_i
- We saw one extra spam email **without** word w_i

Same for ham emails.

$$\mathbb{P}(w_i \mid \text{spam}) = \frac{\left|\text{total spam emails (training set) containing } w_i\right| + 1}{\left|\text{total spam emails (training set)}\right| + 2}$$

$$\mathbb{P}(w_i \mid \text{ham}) = \frac{\left|\text{total ham emails (training set) containing } w_i\right| + 1}{\left|\text{total ham emails (training set)}\right| + 2}$$

Laplace smoothing

Pretend in spam emails (training set):

- We saw one extra spam email **with** word w_i
- We saw one extra spam email **without** word w_i

Same for ham emails.

$$\mathbb{P}(w_i \mid \textbf{spam}) = \frac{\left|\text{total spam emails (training set) containing } w_i\right| + 1}{\left|\text{total spam emails (training set)}\right| + 2}$$

$$\mathbb{P}(w_i \mid \textbf{ham}) = \frac{\left|\text{total ham emails (training set) containing } w_i\right| + 1}{\left|\text{total ham emails (training set)}\right| + 2}$$

$$\mathbb{P}(\text{“buy”} \mid \text{ham}) = \frac{0+1}{2+2} = \frac{1}{4}$$

Example

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$$

$$= \frac{\mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{spam})\ \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"valium"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"valium"}|\text{spam})\mathbb{P}(\text{spam})+\mathbb{P}(\text{"you"}|\text{ham})\mathbb{P}(\text{"buy"}|\text{ham})\mathbb{P}(\text{"valium"}|\text{ham})\mathbb{P}(\text{ham})}$$

Email	**Label**
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5}$$

$$\mathbb{P}(\text{“you”} \mid \text{spam}) =$$

$$\mathbb{P}(\text{“buy”} \mid \text{spam}) =$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) =$$

$$\mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{“you”} \mid \text{ham}) =$$

$$\mathbb{P}(\text{“buy”} \mid \text{ham}) = \frac{0+1}{2+2} = \frac{1}{4}$$

$$\mathbb{P}(\text{"valium"} \mid \text{ham}) =$$

Example

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$$

$$= \frac{\mathbb{P}(\{\text{"you","buy","valium"}\} \mid \text{spam})\ \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you","buy","valium"}\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you","buy","valium"}\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"valium"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"valium"}|\text{spam})\mathbb{P}(\text{spam})+\mathbb{P}(\text{"you"}|\text{ham})\mathbb{P}(\text{"buy"}|\text{ham})\mathbb{P}(\text{"valium"}|\text{ham})\mathbb{P}(\text{ham})}$$

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$$\mathbb{P}(\text{spam}) = \frac{3}{5} \qquad \mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{“you”} \mid \text{spam}) = \frac{1+1}{3+2} = \frac{2}{5} \qquad \mathbb{P}(\text{“you”} \mid \text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

$$\mathbb{P}(\text{“buy”} \mid \text{spam}) = \qquad \mathbb{P}(\text{“buy”} \mid \text{ham}) = \frac{0+1}{2+2} = \frac{1}{4}$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = \qquad \mathbb{P}(\text{"valium"} \mid \text{ham}) =$$

Example

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$$

$$= \frac{\mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{spam})\ \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"valium"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"valium"}|\text{spam})\mathbb{P}(\text{spam})+\mathbb{P}(\text{"you"}|\text{ham})\mathbb{P}(\text{"buy"}|\text{ham})\mathbb{P}(\text{"valium"}|\text{ham})\mathbb{P}(\text{ham})}$$

Email	**Label**
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$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = \frac{3+1}{3+2} = \frac{4}{5} \qquad \mathbb{P}(\text{"valium"} \mid \text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

Example

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$$

$$= \frac{\mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{spam})\ \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"valium"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"valium"}|\text{spam})\mathbb{P}(\text{spam})+\mathbb{P}(\text{"you"}|\text{ham})\mathbb{P}(\text{"buy"}|\text{ham})\mathbb{P}(\text{"valium"}|\text{ham})\mathbb{P}(\text{ham})}$$

$$= \frac{\frac{2}{5}\cdot\frac{2}{5}\cdot\frac{4}{5}\cdot\frac{3}{5}}{\frac{2}{5}\cdot\frac{2}{5}\cdot\frac{4}{5}\cdot\frac{3}{5}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{2}\cdot\frac{2}{5}} \approx 0.7544$$

Email	**Label**
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5} \qquad \mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{“you”} \mid \text{spam}) = \frac{1+1}{3+2} = \frac{2}{5} \qquad \mathbb{P}(\text{“you”} \mid \text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

$$\mathbb{P}(\text{“buy”} \mid \text{spam}) = \frac{1+1}{3+2} = \frac{2}{5} \qquad \mathbb{P}(\text{“buy”} \mid \text{ham}) = \frac{0+1}{2+2} = \frac{1}{4}$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = \frac{3+1}{3+2} = \frac{4}{5} \qquad \mathbb{P}(\text{"valium"} \mid \text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

Example

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$$

$$= \frac{\mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{spam})\,\mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{spam})\,\mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"},\text{"buy"},\text{"valium"}\} \mid \text{ham})\,\mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"}\mid\text{spam})\mathbb{P}(\text{"buy"}\mid\text{spam})\mathbb{P}(\text{"valium"}\mid\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}\mid\text{spam})\mathbb{P}(\text{"buy"}\mid\text{spam})\mathbb{P}(\text{"valium"}\mid\text{spam})\mathbb{P}(\text{spam})+\mathbb{P}(\text{"you"}\mid\text{ham})\mathbb{P}(\text{"buy"}\mid\text{ham})\mathbb{P}(\text{"valium"}\mid\text{ham})\mathbb{P}(\text{ham})}$$

Email	Label
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$$= \frac{\frac{2}{5}\cdot\frac{2}{5}\cdot\frac{4}{5}\cdot\frac{3}{5}}{\frac{2}{5}\cdot\frac{2}{5}\cdot\frac{4}{5}\cdot\frac{3}{5}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{2}\cdot\frac{2}{5}} \approx 0.7544$$

$$\mathbb{P}(\text{spam}) = \frac{3}{5} \qquad \mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{“you”} \mid \text{spam}) = \frac{1+1}{3+2} = \frac{2}{5} \qquad \mathbb{P}(\text{“you”} \mid \text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

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$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = \frac{3+1}{3+2} = \frac{4}{5} \qquad \mathbb{P}(\text{"valium"} \mid \text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

Underflow Prevention

- Multiplication of many probabilities, each of which will be between 0 and 1, can result in floating-point underflow. The product will be too small and will result in arithmetic underflow.

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$$\log(xy) = \log(x) + \log(y)$$

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- Reminder: Log property:

$$\log(xy) = \log(x) + \log(y)$$

- Summing logs of probabilities is better than multiplying probabilities

$$\log\left(\prod_{i=1}^{n} p_i\right) = \log(p_1 p_2 \dots p_n) = \log(p_1) + \log(p_2) + \dots + \log(p_n)$$
$$= \sum_{i=1}^{n} \log(p_i)$$

Applying underflow prevention

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\ \mathbb{P}(\text{spam})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$

$$\mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\ \mathbb{P}(\text{ham})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$

We will output **spam** iff:

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) > \mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\})$$

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$$\mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\ \mathbb{P}(\text{ham})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$

Denominators are equal and cancel when comparing

We will output **spam** iff:

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) > \mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\})$$

$$\iff \mathbb{P}(\{w_1, w_2, \ldots, w_n\} \mid \text{spam})\mathbb{P}(\text{spam}) > \mathbb{P}(\{w_1, w_2, \ldots, w_n\} \mid \text{ham})\mathbb{P}(\text{ham})$$

Applying underflow prevention

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\, \mathbb{P}(\text{spam})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\, \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\, \mathbb{P}(\text{ham})}$$

$$\mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\, \mathbb{P}(\text{ham})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\, \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\, \mathbb{P}(\text{ham})}$$

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$$\iff \mathbb{P}(w_1 \mid \text{spam})\mathbb{P}(w_2 \mid \text{spam}) \cdots \mathbb{P}(w_n \mid \text{spam})\mathbb{P}(\text{spam}) > \mathbb{P}(w_1 \mid \text{ham})\mathbb{P}(w_2 \mid \text{ham}) \cdots \mathbb{P}(w_n \mid \text{ham})\mathbb{P}(\text{ham})$$

Applying underflow prevention

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\}|\text{ spam})\,\mathbb{P}(\text{spam})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\}|\text{ spam})\,\mathbb{P}(\text{spam})+\mathbb{P}(\{w_1,w_2,\ldots,w_n\}|\text{ ham})\,\mathbb{P}(\text{ham})}$$

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We will output **spam** iff:

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$$\iff \mathbb{P}(\{w_1, w_2, \ldots, w_n\} \mid \text{spam})\mathbb{P}(\text{spam}) > \mathbb{P}(\{w_1, w_2, \ldots, w_n\} \mid \text{ham})\mathbb{P}(\text{ham})$$

$$\iff \mathbb{P}(w_1 \mid \text{spam})\mathbb{P}(w_2 \mid \text{spam}) \cdots \mathbb{P}(w_n \mid \text{spam})\mathbb{P}(\text{spam}) > \mathbb{P}(w_1 \mid \text{ham})\mathbb{P}(w_2 \mid \text{ham}) \cdots \mathbb{P}(w_n \mid \text{ham})\mathbb{P}(\text{ham})$$

Taking the log of two sides:

$$\iff \log(\mathbb{P}(\text{spam}))+\sum_{i=1}^{n}\log(\mathbb{P}(w_i \mid \text{spam}) > \log(\mathbb{P}(\text{ham}))+\sum_{i=1}^{n}\log(\mathbb{P}(w_i \mid \text{ham})$$

Summary: Naive Bayes Algorithm steps

1. **TRAINING**

1.1. Compute the proportion of emails in the **training set** that is spam or ham:

$$\mathbb{P}(\text{spam}) = \frac{\text{total spam emails (in training set)}}{\text{total emails (in training set)}} \qquad \mathbb{P}(\text{ham}) = \frac{\text{total ham emails (in training set)}}{\text{total emails (in training set)}}$$

1.2. Iterate over the **training set**, for each unique word **x**, count:

- How many **spam emails** in the training set contain **x**
- How many **ham emails** in the training set contain **x**

Summary: Naive Bayes Algorithm steps

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1.2. Iterate over the **training set**, for each unique word **x**, count:

- How many **spam emails** in the training set contain **x**
- How many **ham emails** in the training set contain **x**

2. **TESTING**

Iterate over the **test set**, for each unlabelled email **D**:

- Create a set **S** of **n** unique words appearing in **D**:$\{w_1, w_2, \ldots, w_n\}$
- For each word w_i in set **S**, calculate:

$$\mathbb{P}(w_i \mid \text{spam}) = \frac{\left|\text{total spam emails (training set) containing } w_i\right| + 1}{\left|\text{total spam emails (training set)}\right| + 2} \qquad \mathbb{P}(w_i \mid \text{ham}) = \frac{\left|\text{total ham emails (training set) containing } w_i\right| + 1}{\left|\text{total ham emails (training set)}\right| + 2}$$

  - Note: If word w_i doesn't appear in the training set, we still calculate the above probabilities, with:

$$\left|\text{total spam emails (training set) containing } w_i\right| = \left|\text{total ham emails (training set) containing } w_i\right| = 0$$

- If $\log(\mathbb{P}(\text{spam})) + \sum_{i=1}^{n} \log(\mathbb{P}(w_i \mid \text{spam})) > \log(\mathbb{P}(\text{ham})) + \sum_{i=1}^{n} \log(\mathbb{P}(w_i \mid \text{ham}))$

  Predict email **D** as **spam**

  Otherwise, predict email **D** as **ham**


SPAM®
Classic
Ingredients: Pork with Ham, Salt, Water, Modified Potato Starch, Sugar, Sodium Nitrite
Serving Suggestion
NET WT 12 OZ (340g)
Glorious SPAM®

Alex Tsun