CSE 312
Foundations of Computing II

Aleks Jovcic

Welcome to summer quarter!

Hi! :)

https://courses.cs.washington.edu/312/22su

Agenda

- **Course Overview**
  - Introductions
  - Course Content
  - Administrivia
- **Intro to Counting**
  - Sum Rule
  - Product Rule
  - Permutations
  - Complimentary Counting

Your Staff!

Aleks Jovcic
(he/him)

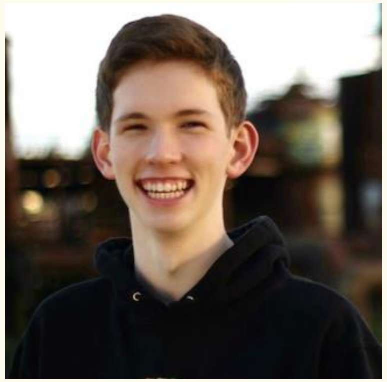
Your Instructor!

Just graduated with my bachelor's degree in Computer Science!

Jinghua Sun (Head TA)
(she/her)

Arya GJ
(he/him)

Lukshya Ganjoo
(he/him)



Elliott Zackrone
(he/him)

Xinyue Chen
(she/her)

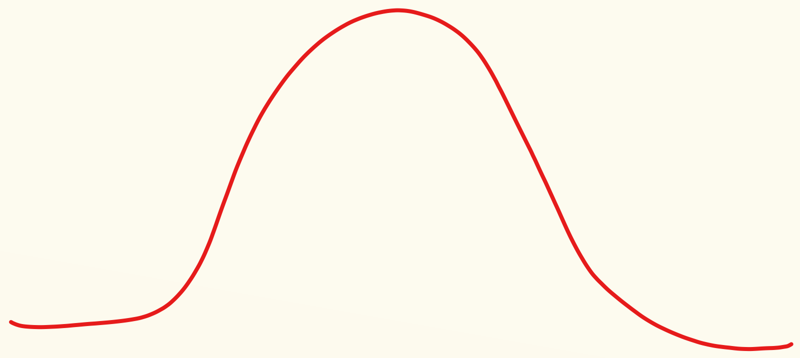
Abbey Regan
(she/her)

Course Content

- **Probability and Statistics for Computer Scientists**
  - Foundation of several CS Topics
  - Establishing the fundamentals
- **Context for the math**
  - Technical applications (coding)
  - Real-world implications and assumptions
- **Practice for higher-level courses**

Course Roadmap

- **Counting (Combinatorics)** ← we are here
  - Week 1-2
- **Probability**
  - Week 2-3
- **Random Variables**
  - Week 4-5
- **Multiple Random Variables**
  - Week 6
- **The Normal Random Variable**
  - Week 7
- **Statistics**
  - Week 8



Syllabus Overview

Found in full on course website

CSE 312

Foundations of Computing II

Lecture 1: Counting I

PAUL G. ALLEN SCHOOL
OF COMPUTER SCIENCE & ENGINEERING

Aleks Jovcic

Slide Credit: Based on Anna Karlin's slides for 312 21au

Today: Counting

Prerequisite: Set Theory

A set S ~~set~~ is an unordered collection of objects with no duplicates. They can be finite or infinite.

$S \subseteq A$ i

The cardinality of S is denoted $|S|$, which is the number of elements in the set.

$\phi = \{\}$

$S = \{3 \quad 18 \quad 20091\}$

Examples: $S = \{3, 18, 20091\}$

$S = \{apple, orange\}$

$S = \{★, ♠\}$

S = all positive integers

We are interested in counting the number of elements with a certain given property.

$S \subseteq U$

$|S$

"How many ways are there to assign 7 TAs to 5 sections, such that each section is assigned to two TAs, and no TA is assigned to more than two sections?"

"How many integer solutions $(x, y, z) \in \mathbb{Z}^3$ does the equation $x^3 + y^3 = z^3$ have?"

Generally: Question boils down to computing cardinality $|S|$ of some given set S.

(Discrete) Probability and Counting are Twin Brothers

"What is the probability that a random student from CSE312 has black hair?"

$|S|$ $\quad S \subseteq U$

$$= \frac{\#\,students\ with\ black\ hair}{\#students}$$

$|U|$

Sum Rule

If elements of your set can be from

- Either one of n options,
- OR one of m options with NO overlap with the previous n,

then the number of possible outcomes is

$$n + m$$

Counting lunches

$6+8=14$

$8+6=14$

If your lunch can be **either** one soup (6 choices) **or** one salad (8 choices), how many possible lunches?

6 5 15

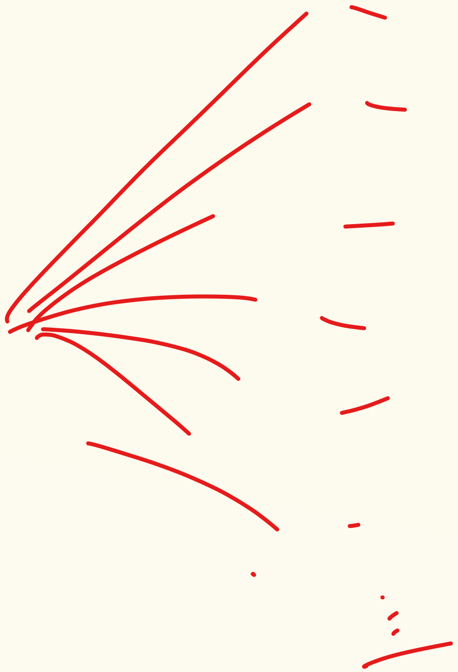
$\boxed{14}$

8

Product Rule: If each element is constructed by a sequential process where there are

- n_1 choices for the first step,
- n_2 choices for the second step (given the first choice), … , and
- n_k choices for the k^{th} step (given the previous choices),

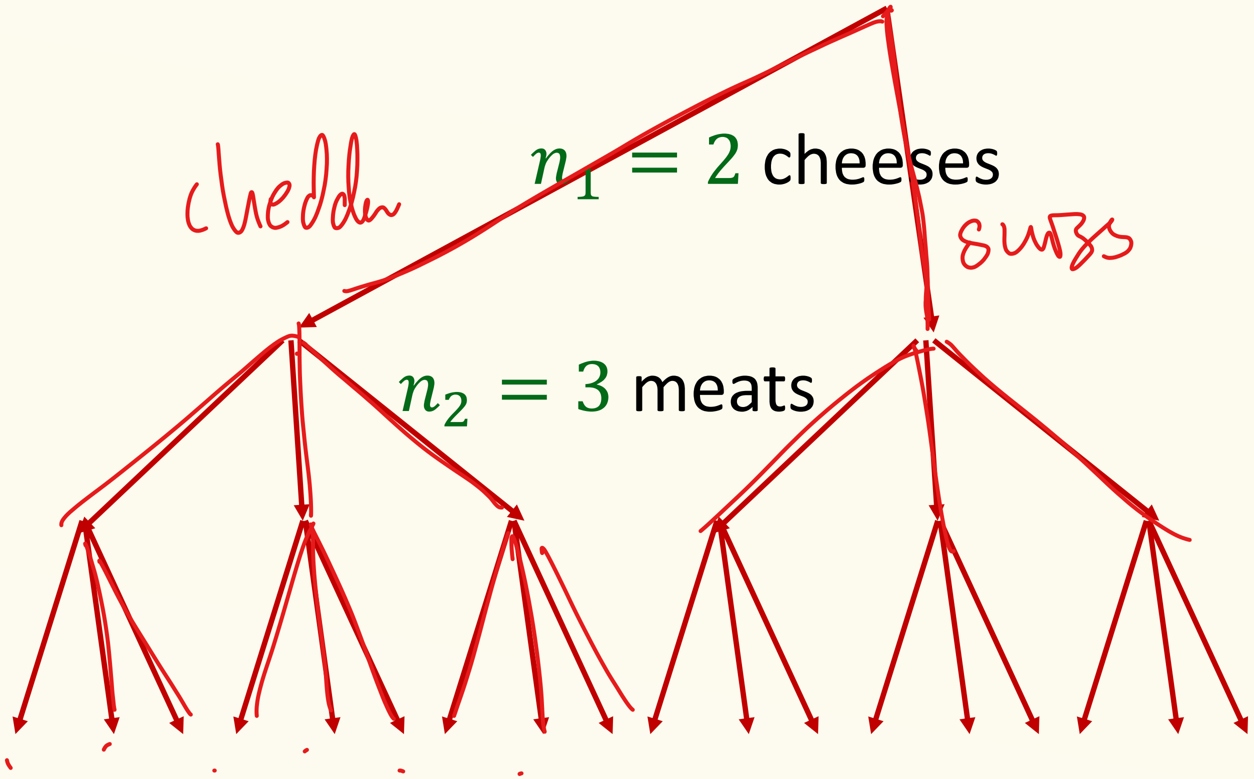
then the total number of possibilities is $n_1 \times n_2 \times \cdots \times n_k$



Product Rule: In a sequential process, if there are

- n_1 choices for the first step,
- n_2 choices for the second step (given the first choice), …, and
- n_k choices for the k^{th} step (given the previous choices),

then the total number of possibilities is $n_1 \times n_2 \times \cdots \times n_k$



$n_1 = 2$ cheeses

$n_2 = 3$ meats

$n_3 = 3$ veggies

Example: "How many subways?"

$\boxed{2} \times \boxed{3} \times \boxed{3} = \boxed{18}$

Example – Strings

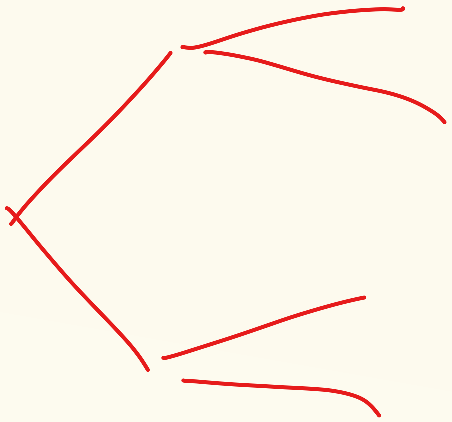
5

How many binary strings of length n over the alphabet $\{0,1\}$?

3

- E.g., $0 \cdots 0$, $1 \cdots 1$, $0 \cdots 01$, …

$$\boxed{2} \times \boxed{2} \times \boxed{2} \times \boxed{2} \times \boxed{2} = \boxed{2^n}$$



Example – Strings

How many strings of length 5 over the alphabet $\{A, B, C, \ldots, Z\}$ are there?

AAAAA XYABT

- E.g., AZURE, BINGO, TANGO, STEVE, SARAH, …

$\boxed{26} \times \boxed{26} \times \boxed{26} \times \boxed{26} \times \boxed{26} = \boxed{26^5}$

Example – Power set

Definition. The **power set** of S is

$$2^S \stackrel{\text{def}}{=} \{X : X \subseteq S\}$$

Example.

$S = \{★, ♠\}$ $\quad$ $2^{\{★, ♠\}} = \{\emptyset, \{★\}, \{♠\}, \{★, ♠\}\}$

$S = \emptyset$ $\quad$ $2^{\emptyset} = \{\emptyset\}$

…

How many different subsets of S are there if $|S| = n$?

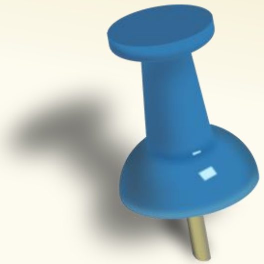
Example – Power set – number of subsets of S

$$S = \{e_1, e_2, e_3, \cdots, e_n\}$$

What is the number of subsets of S,   i.e., $|2^S|$?

$$\square \times \square \times \square \times \ldots \times \square = \square$$

Example – ATMs and Pin codes




- How many 4 –digit pin codes are there?
- Each digit one of {0, 1, 2,…, 9}

$$\boxed{10} \times \boxed{10} \times \boxed{10} \times \boxed{10} = \boxed{10^4}$$

# possible **first** digits	# possible **second** digits	# possible **third** digits	# possible **fourth** digits	# possible **pins**

Example – ATMs and Pin codes – Stronger Pins

- How many **10–digit** pin codes are there with no repeating digit?
- Each digit one of {0, 1, 2,… , 9}; must use each digit ***exactly once***

$$\boxed{10} \times \boxed{9} \times \boxed{8} \times \ldots \times \boxed{2} \times \boxed{1} = \boxed{10!}$$

# possible **first** digits	# possible **second** digits	# possible **third** digits	…		# possible **pins**

Permutations

"How many ways to order n distinct objects?"

Answer = $n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$

Definition. The **factorial function** is

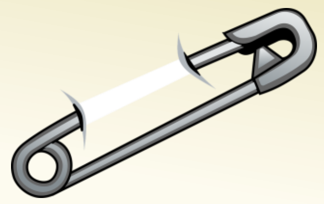
$$n! = n \times (n-1) \times \cdots \times 2 \times 1$$

Read as ``n factorial"

Note: $0! = 1$

Huge: Grows exponentially in n

Example – ATMs and Pin codes – Tricky Pins



- How many 10–digit pin codes with **at least one digit repeated once**?
- Examples: 1111111111, 1234567889, 1353483595

1 1 8 9

10 x 10 x 10 x … x 10 x 9 =

possible **first** digits

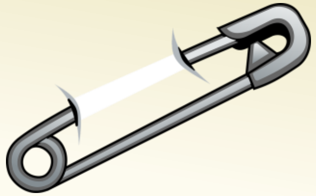
possible **second** digits

possible **third** digits

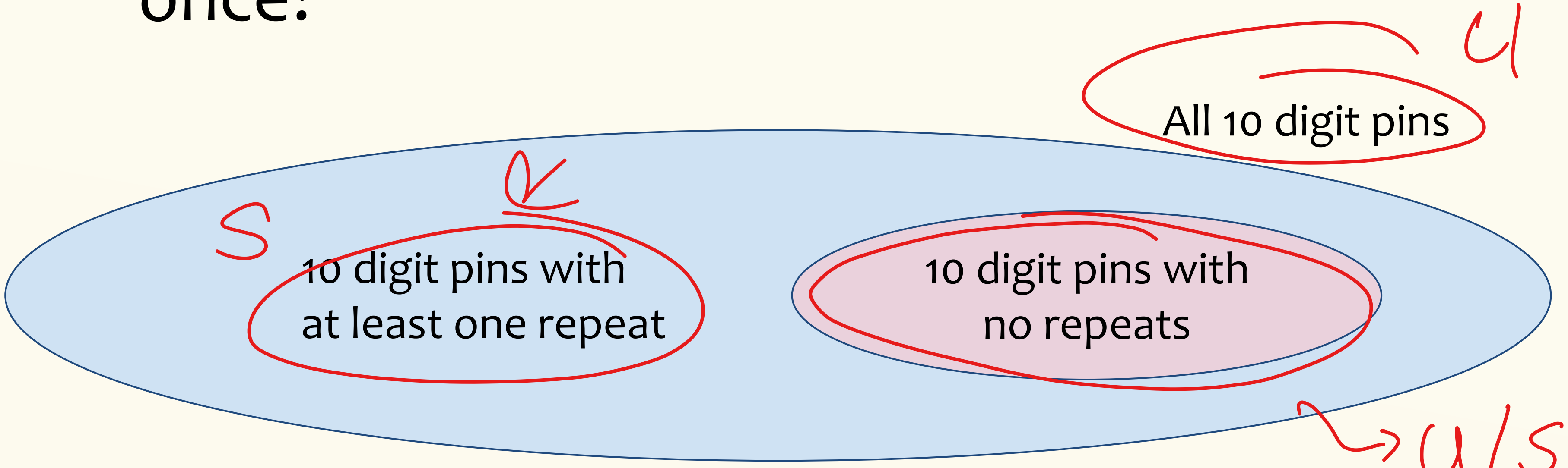
…

possible **pins**

Example – ATMs and Pin codes – Tricky Pins



- How many 10–digit pin codes with at least one digit repeated once?



$S \subseteq U$

$U \setminus S = S^c$

$$|S| + |S^c| = |U|$$

Complementary Counting

Let U be a set and S a subset of interest

Let U \S denote the set difference (the part of U that is not in S)

$|S| = |U| - |U/S|$

$= \boxed{10^{10} - 10!}$

Then $|U \setminus S| = |U| - |S|$

And $|S| = |U| - |U/S|$

Quick Summary

- Sum Rule

  If you can choose from

  – Either one of n options,

  – OR one of m options with NO overlap with the previous n,

  then the number of possible outcomes of the experiment is $n + m$

- Product Rule

  In a sequential process, if there are
  - n_1 choices for the first step,
  - n_2 choices for the second step (given the first choice), …, and
  - n_k choices for the k^{th} step (given the previous choices),

  then the total number of outcomes is $n_1 \times n_2 \times \cdots \times n_k$

Quick Summary

- Complementary Counting:
- Instead of counting |S|, count |U| - |U/S|
- Permutations: How many ways to uniquely order n distinct elements?
  - Product rule → n!

The first concept check (CC) will be out at 2PM and is due 11:30AM Friday

The concept checks are meant to help you immediately reinforce what is learned in each lecture.
Students from previous quarters found them really useful!

Pset 1 is out now! Due Friday, July 1st at 11:59pm PST

First problem set is a bit shorter than future ones.

Includes some prerequisite review and will onboard you with Python.