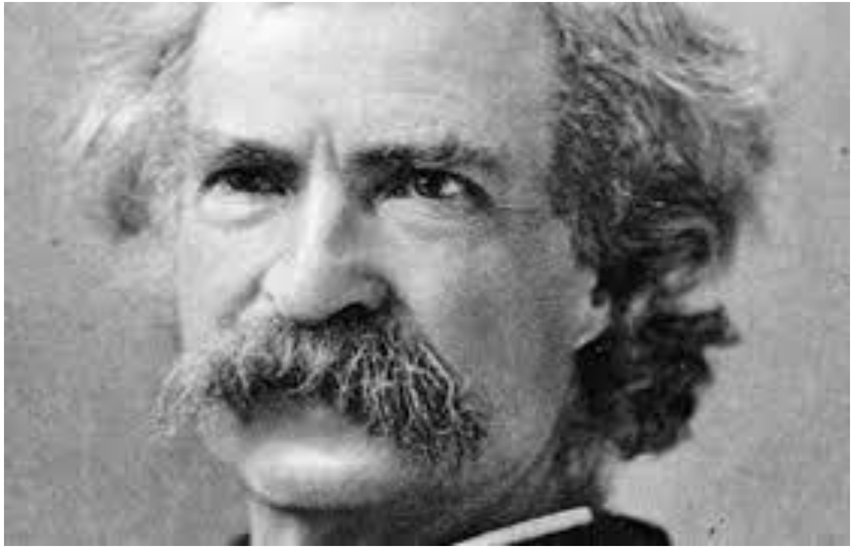
Random Quote

“There are three kinds of lies: lies, damned lies, and statistics.”

- Mark Twain



CSE 312

Foundations of Computing II

Lecture 29: How to lie/be misled/detect lies with statistics

PAUL G. ALLEN SCHOOL OF COMPUTER SCIENCE & ENGINEERING

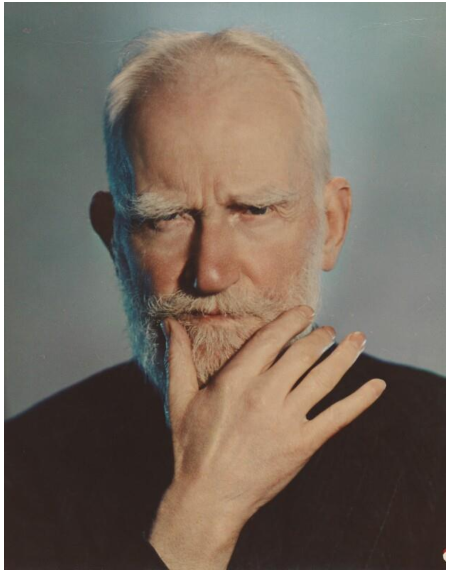
Anna R. Karlin

Slide Credit: Based on Stefano Tessaro's slides for 312 19au incorporating ideas from Alex Tsun, Maya Bar-Hillel & myself ☺

Random Quote

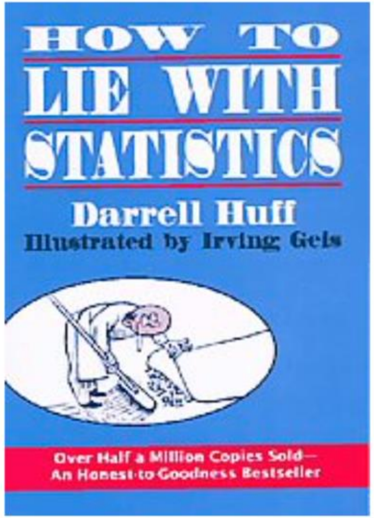
“It is the mark of a truly intelligent person to be moved by statistics”

- George Bernard Shaw



The Book

- Published in 1954, over 500,000 copies sold
- “A great introduction to the use of statistics, and a great refresher for anyone who’s already well versed in it” - Bill Gates.



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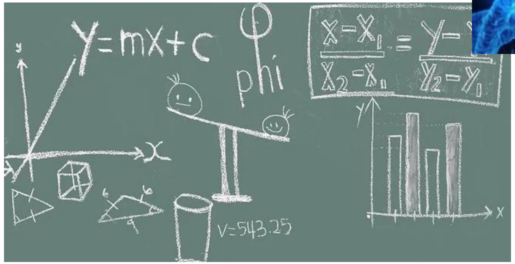
- Doesn’t teach how to lie with statistics, but how we are/can be lied to using statistics
- In the current age, we are lied to all the time, e.g., by **politicians**, and **marketers**.
    - Often make decisions based on these lies: “4 out of 5 dentists recommend….”

To be clear...

- Many lies are unintentional
- People passing on misinformation/bad information that they don't even know is bad.
- People using bad data to make inferences
- People not understanding statistics well enough

What is “Statistics”?

- A way to make sense of information from data
- Framework for thinking, for reaching insights, and solving problems.
- Numbers alone mean very little without context
- Statistics is a marriage of:
  - Math
  - Science
  - Art



Random Quote

“Statistical Thinking will one day be as necessary for efficient citizenship as the ability to read and write”

- H.G. Wells

Statistical Inference

- Making an estimate or prediction about a **population** based on a **sample**.

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Statistical Inference

- Making an estimate or prediction about a **population** based on a **sample**.
  - Often very expensive/impossible to survey an entire population (all students at UW, all residents in the U.S)
  - Need to use a **random unbiased** *sample* of *population* to draw conclusions (with some chance/margin of error)

Sampling Gone Wrong (Bias)

“The Literary Digest” Magazine wanted to predict 1936 election:

- Alfred Landon vs Franklin D Roosevelt
- Sent 10 million surveys and received 2.4 million responses
- From a “List” containing: their subscribers, owners of cars and telephones

Electoral Votes	**Prediction**	**Actual**
Landon		
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Electoral Votes	**Prediction**	**Actual**
Landon	370	
Roosevelt	161	

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Electoral Votes	**Prediction**	**Actual**
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Roosevelt	161	523

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What went wrong?

Sampling Gone Wrong (Bias)

Let $x_1, x_2, \ldots, x_n$ be **iid** samples…

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Let $x_1, x_2, \ldots, x_n$ be **iid** samples...

- Not Representative
    - Voluntary Response Bias
        - Only 24% of respondents answered the poll.

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- Not Random
  - Convenience Sampling
    - Only to people whose contact information they have.
    - Like standing outside a church and asking “Do you believe in God?”, using those samples to represent the US population.

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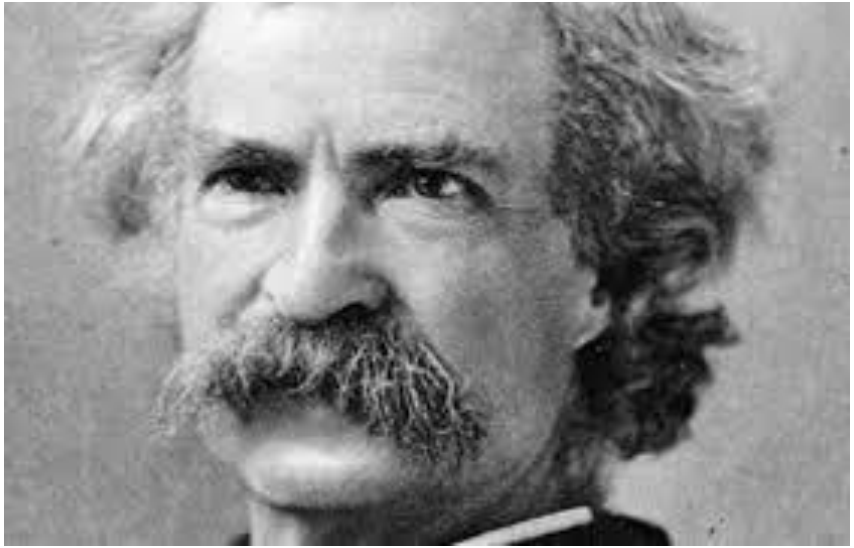
    - Like standing outside a church and asking "Do you believe in God?", using those samples to represent the US population.

More samples is NOT a solution for bad sampling technique...

Random Quote

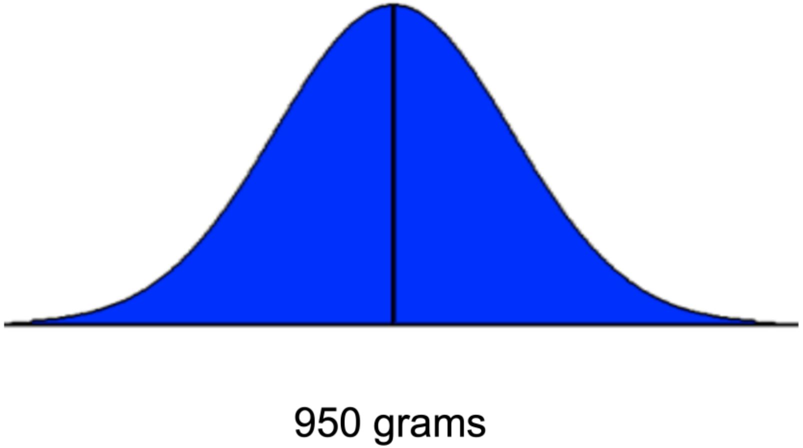
“Facts are stubborn, but statistics are more pliable.”

- Mark Twain



Detecting lies with statistics

A story about the famous French mathematician Henri Poincare



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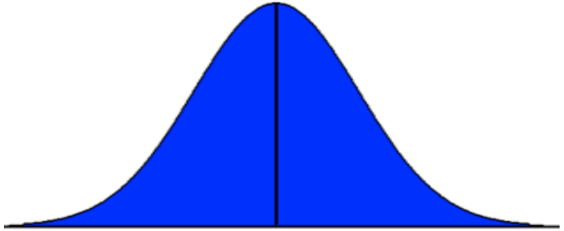
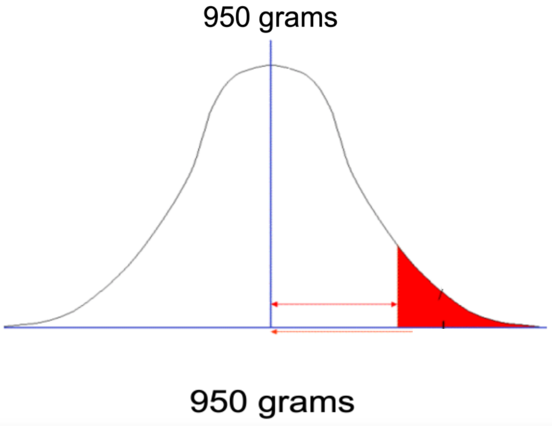




To fake a distribution…

You’d better know what it looks like….

People that are untrained in statistics often don’t.

For example, people are really bad at faking a sequence of fair coin tosses.

Random Quote

“It’s easy to lie with statistics. It’s hard to tell the truth without statistics.”

- Andrejs Dunkels

First digit phenomenon

Suppose that I pick a random integer in the range 1..999

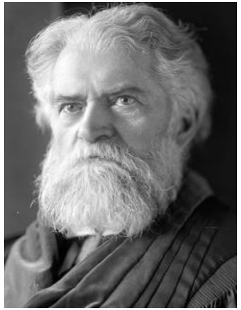
What's the chance that the first digit of the number I pick is a 1?

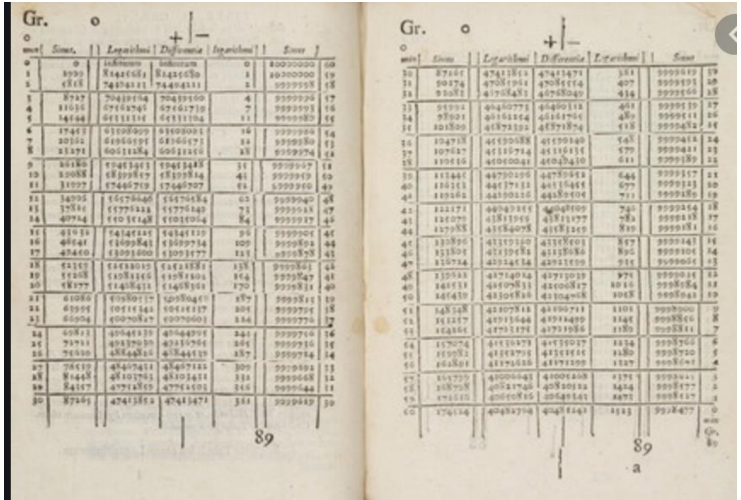
a). About 1/9
b). About 11%
c) 30%
d) I don't know.

Benford’s Law

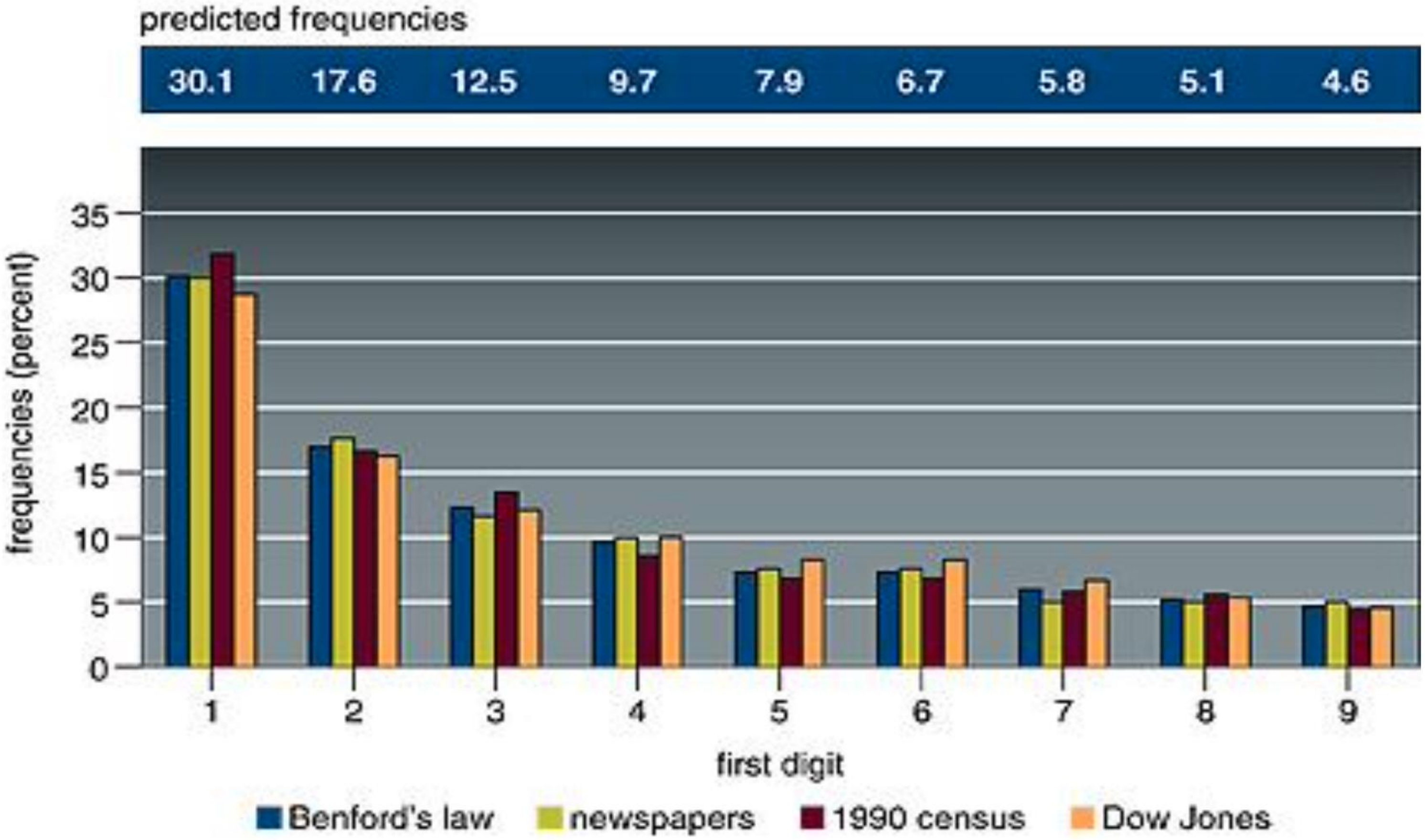
How about in real life? Do certain digits in numbers collected randomly from the front pages of the newspaper or census statistics or from stock-market prices occur more often than others?

Frequency with which first significant digit is d = log (1 + 1/d)





From "The First-Digit Phenomenon" by T. P. Hill, American Scientist, July-August 1998)



Long-term efforts to “prove” Benford’s Law

Properties of a random sample that result in such a distribution? E.g. not true for Unif {1,...999}

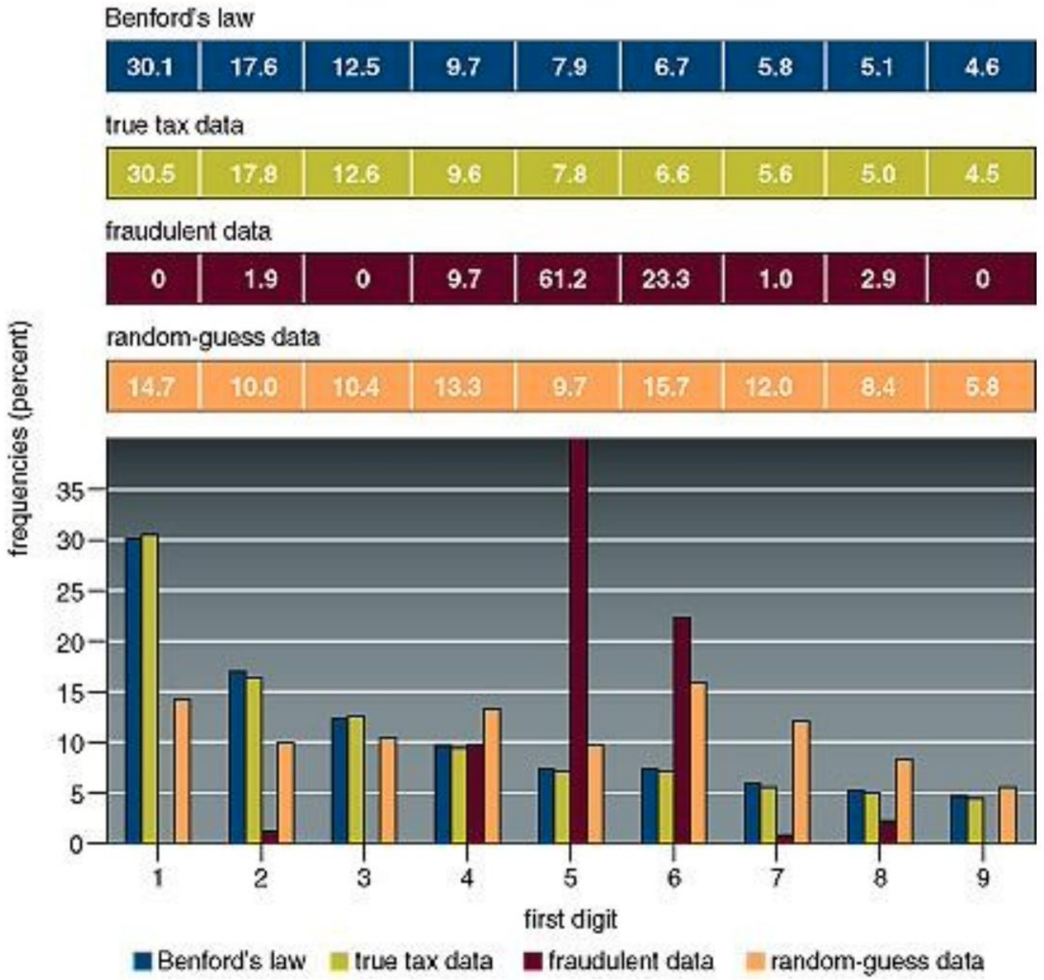
- Scale invariance: e.g. convert from dollars to pesos shouldn’t change the first digit frequencies much
- Independent of base: Equally valid when numbers expressed in base 10, base 100, or others

The only distributions on numbers that satisfies these conditions satisfy

Pr(first significant digit = d) = log (1 + 1/d)

Modern Application

- Using Benford’s law to detect fraud or fabrication of data in financial documents.



Random Quote

“It is easy to lie with statistics, but easier to lie without them”.

Fred Mosteller

“Too good to be true”

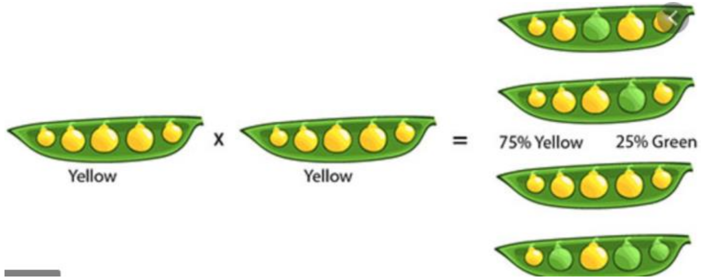
- The special case of not appreciated the expected magnitude of sampling error.

- Data comes out ”too good to be true”, a telltale sign of having been tampered with, if not generated out of whole cloth.

Gregor Mendel’s Sweet Peas

Postulated that self fertilization Of hybrid yellow-seeded sweet peas would yield offspring with

- 0.75 chance yellow-seeded
- 0.25 chance green seeded.



1865, reported results of 8023 experiments:

- 0.7505 yellow-seeded
- 0.2495 green-seeded.

Probability of observations as close to expected value as he reported is minute.

Some telltale signs of fakery....

- Wrong shape
- Too close to expected value (especially replicated)
- Too far from expected value
- Replications too good to be true.

Another famous example: Sir Cyril Burt’s Twins

3 data sets: same to 3 decimal points.

Random Quote

"82.123456789% of statistics are made up."

- Alex Tsun

p-Hacking

Manipulating data or statistical analyses to get **“significant p-values”**

First, a brief primer on hypothesis testing and p-values.

Suppose that I believe that jelly beans cause acne.
How might I provide evidence of this?
Approach – “probabilistic proof by contradiction”

Hypothesis Testing

Want to provide evidence that the null hypothesis can be rejected!

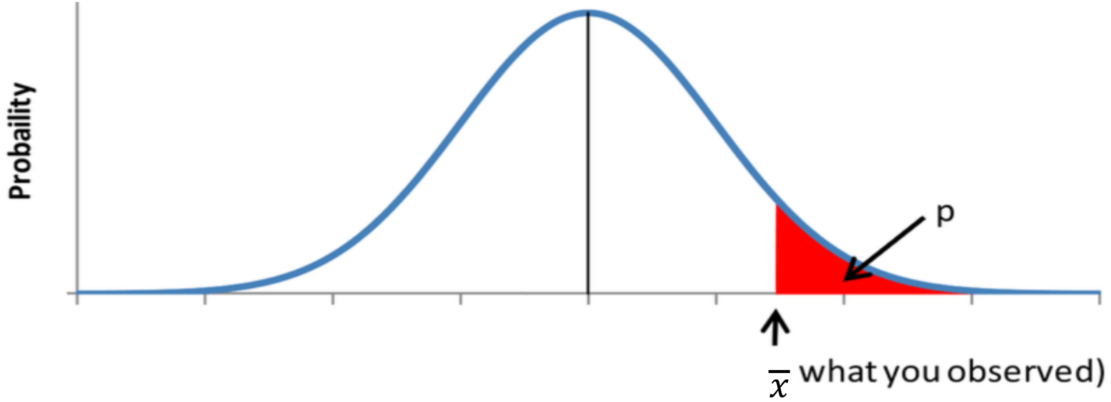
Average teenager has amount of acne with mean μ and variance σ^2

H_0 – null hypothesis (baseline): the mean amount of acne someone who eats jelly beans has is μ, i.e., **jelly beans have no effect on acne**

H_A - Alternative hypothesis: the mean amount of acne someone who eats jelly beans has is $> \mu$

Choose ***significance level,*** say 0.05

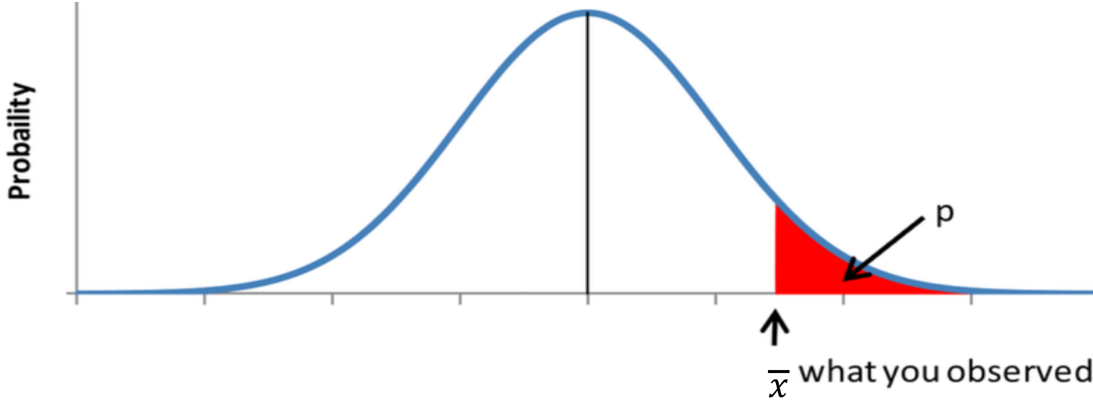
Observe 100 jelly-bean-eating teenagers and measure their acne levels.
Suppose sample mean observed $\overline{x}$



Hypothesis Testing

H_0 – null hypothesis (baseline):
jelly beans have no effect on acne

H_A - Alternative hypothesis:
Jelly beans increase acne



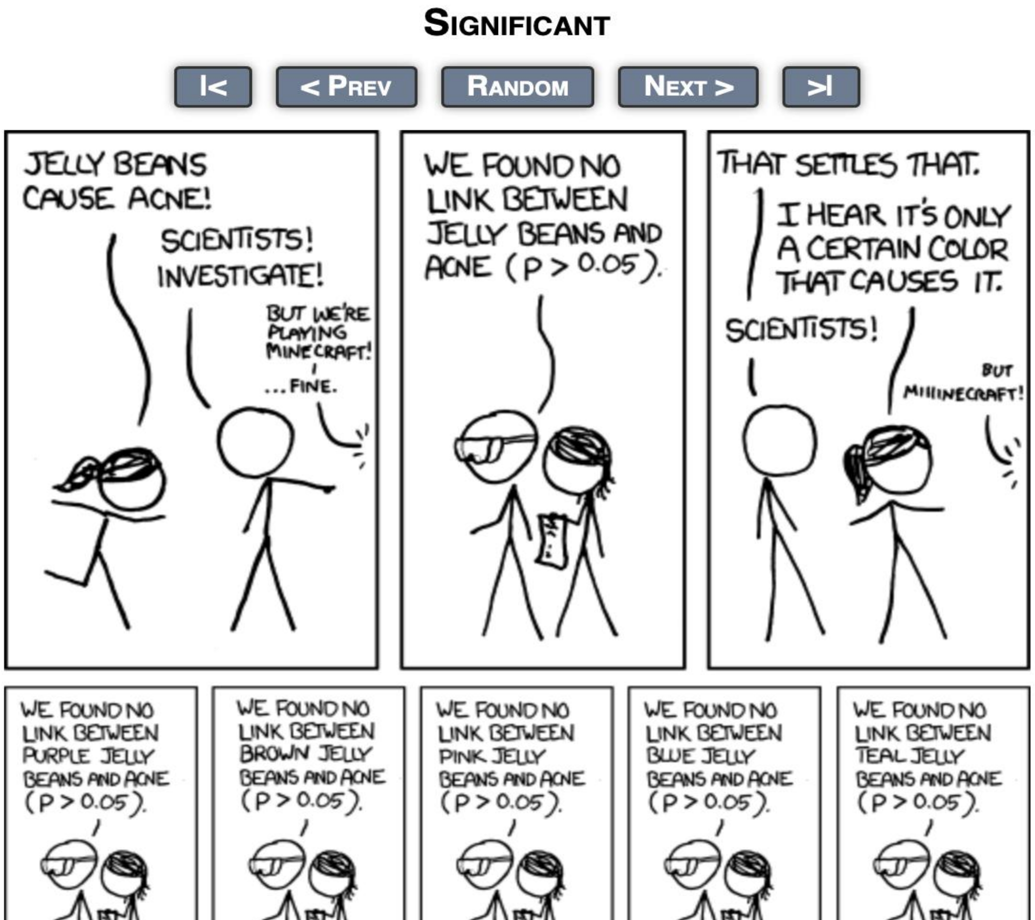
Suppose find that for measured $\bar{x}$

Pr (observing amount of acne this high if H_0 true) = **Pr ($\bar{X} \geq \bar{x}$) = 0.0162**. This is our **p-value.**

If $p < 0.05$ reject H_0 at the 0.05 significance level, i.e., **strong statistical evidence that jelly beans cause an increase in acne.** (If H_0 was true, this would be a very unlikely outcome).

If $p > 0.05$, fail to reject $H_{0;}$
Not enough evidence to suggest the jelly bean effect on acne was significant.

Hacking


SIGNIFICANT
|<
< PREV
RANDOM
NEXT >
>|
JELLY BEANS CAUSE ACNE!
SCIENTISTS! INVESTIGATE!
BUT WE'RE PLAYING MINECRAFT!
...FINE.
WE FOUND NO LINK BETWEEN JELLY BEANS AND ACNE (P > 0.05).
THAT SETTLES THAT.
I HEAR IT'S ONLY A CERTAIN COLOR THAT CAUSES IT.
SCIENTISTS!
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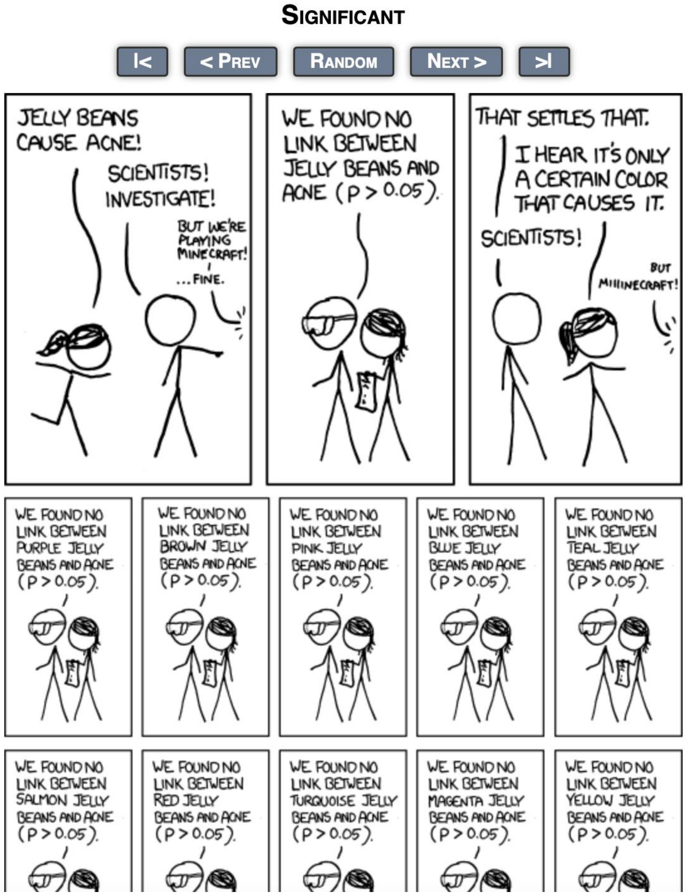
p-Hacking



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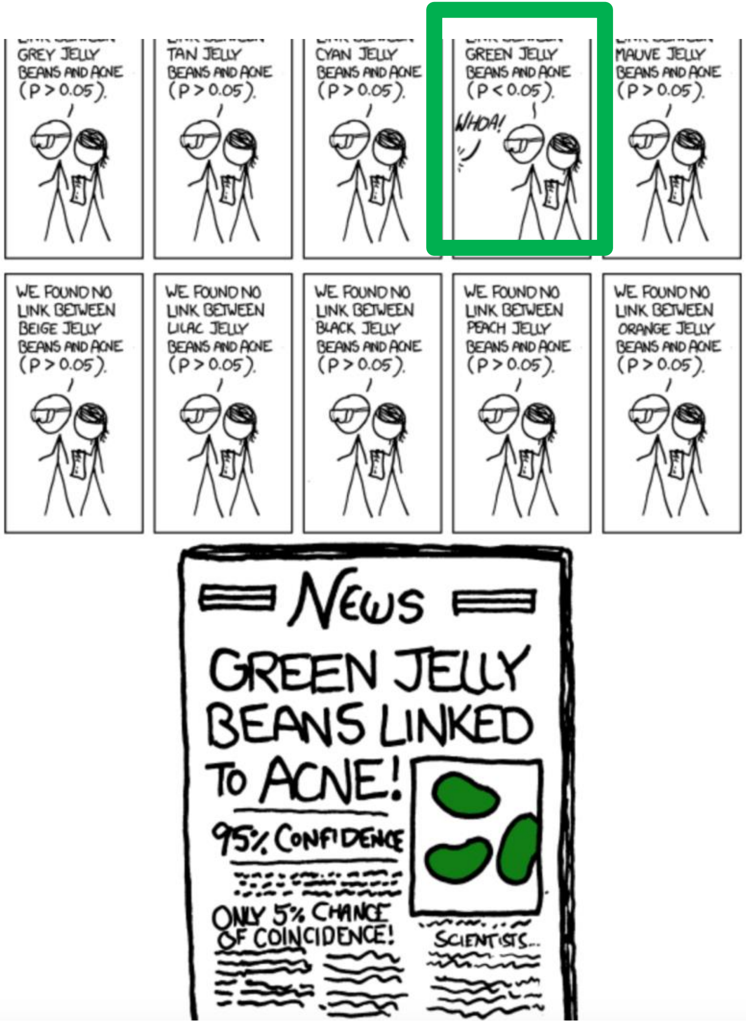
WE FOUND NO LINK BETWEEN SALMON JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN RED JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN TURQUOISE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN MAGENTA JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN YELLOW JELLY BEANS AND ACNE (P > 0.05).



GREY JELLY BEANS AND ACNE (P > 0.05).
TAN JELLY BEANS AND ACNE (P > 0.05).
CYAN JELLY BEANS AND ACNE (P > 0.05).
GREEN JELLY BEANS AND ACNE (P < 0.05).
WHOA!
MAUVE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BEIGE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN LILAC JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BLACK JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN PEACH JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN ORANGE JELLY BEANS AND ACNE (P > 0.05).
NEWS
GREEN JELLY BEANS LINKED TO ACNE!
95% CONFIDENCE
ONLY 5% CHANCE OF COINCIDENCE!
SCIENTISTS...

p-Hacking

- Scientists concluded that “Eating green jelly beans gives you more acne” after testing that teenagers who ate green jelly beans have more acne than those who don’t, with a p-value of 0.05”.
    - **The p-value means:** if the null hypothesis is true (teens who eat green jelly beans and those who don’t have the **same** amount of acne), the probability of observing at least as extreme an outcome as we did is p.
    - **Putting it another way, a p-value of 0.05 means:** only a 5% chance of seeing this much acne if green jelly beans don’t cause acne
    - But what if I repeat the experiment 20 times?
    - The chance that in 20 trials I will never get a p value < 0.05 is

$$0.95^{20} \approx 0.358$$

In other words 64% of the time one of these tests will be significant.

This result has no significance! Happened by random chance!

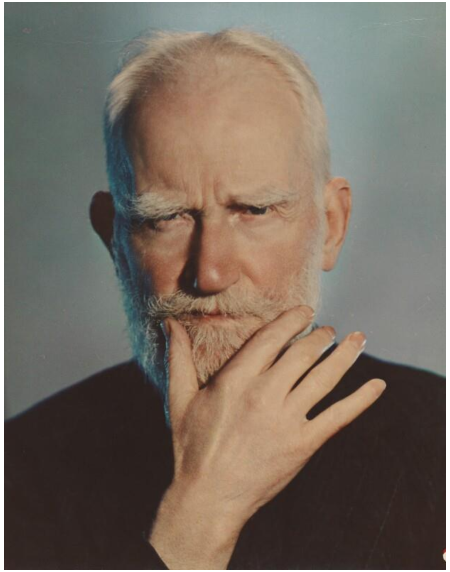
p-Hacking

- **<u>Definition</u>**: Performing the same hypothesis test multiple times in order to get a statistically significant result.
- The particularly evil thing: reporting only the significant tests, but not reporting the other 19 tests.....

Random Quote

"If at first you don't succeed, try two more times so your failure is statistically significant".

- George Bernard Shaw



Random Quote

“Torture numbers, and they’ll confess to anything”

- George Easterbrook

Another interesting misuse of statistics

Attali/Bar-Hillel noticed that SAT answer keys are not randomized.

Keys are balanced rather than randomized.

Was easy for statisticians to detect by examining published tests.

This is a case of thinking "**randomization is too important to be left to chance** "!

Suggests a strategy for test-takers

- Answer all the questions you can.
- When guessing the rest, pick an answer position that occurs least frequently in your answers.

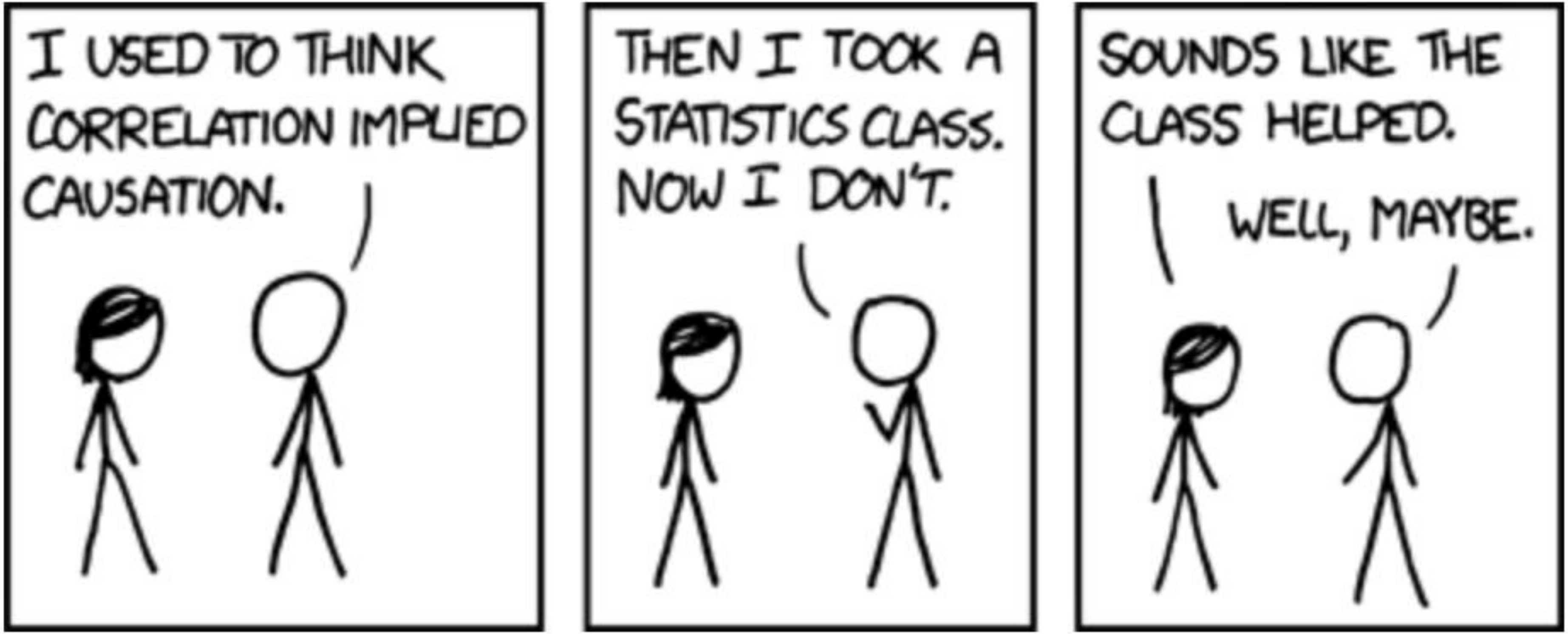
Simulations shows this adds 10-16 points over random guessing.

Claimed to be more gain than some very expensive SAT prep courses!

Conclusions

1. Determine if the samples are **random** and **representative**.
2. Ask for a confidence interval.
3. Be dubious. Be extremely dubious.
4. Don’t make up data or statistics. You’ll get caught.
5. Be wary of p-hacking (and don’t do it yourself)!
6. Be careful about seeing patterns where there are none.
7. Correlation does not imply causation.

Random Quote

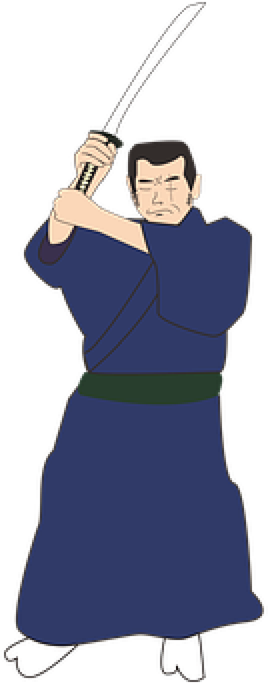


Source: https://xkcd.com/552/

Random Quote

"Data is the sword of the 21st century, those who wield it well, the Samurai."

- Jonathan Rosenberg (ex-Google SVP)



Random Quote

“Do not trust any statistics you did not fake yourself”

- Winston Churchill

Staring Down a Statistic

1. Who says so?
2. How do they know this is true?
3. What’s missing?
4. Did somebody change the subject?
5. Does it make sense?

Correlation → Causation?

- “People who use Senserdime generally have less cavities than those who use generic brands”.

Correlation → Causation?

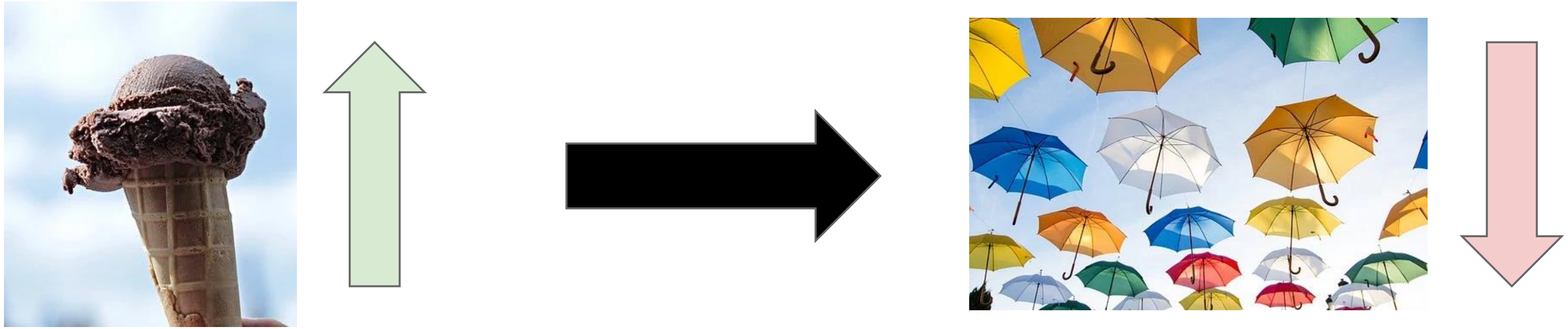
- “People who use Senserdime generally have less cavities than those who use generic brands”.
    - Even if we had a stat-sig p-value (and rejected H_0), correlation does not imply causation.
    - Cannot say “Senserdime prevents cavities”.

Correlation → Causation?

- "People who use Senserdime generally have less cavities than those who use generic brands".
  - Even if we had a stat-sig p-value (and rejected H_0), correlation does not imply causation.
  - Cannot say "Senserdime prevents cavities".
  - Turns out, Senserdime costs $120,000 per tube. This means only wealthy people can afford it. Wealthy people often have access to good healthcare (e.g., dentists). Senserdime didn't do anything!

Correlation → Causation?

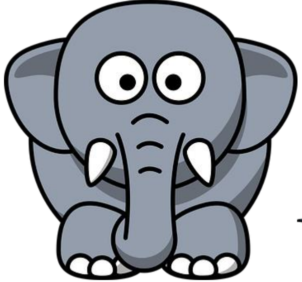
- “When ice cream sales go up, umbrella sales go down.”



Correlation → Causation?

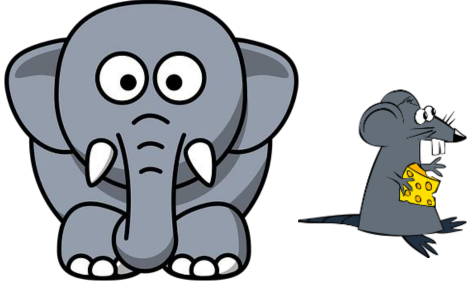
- "When ice cream sales go up, umbrella sales go down."
  - Both generally happen when the weather is sunny.
  - Ice cream sales rise did not CAUSE umbrella sales to go down. The weather CAUSED both of these things to happen.
  - Again, correlation does not imply causation!

Size-Based Sampling




- Let’s say there are 100 families. 50 families have five children each, and 50 families only have a single child.

Size-Based Sampling



- Let’s say there are 100 families. 50 families have five children each, and 50 families only have a single child.
  - What is the expected number of ***siblings*** a random child has?
    - **Choices**: 0, 1, 2, 2.5, 3, 3.33, 4

Poll 5

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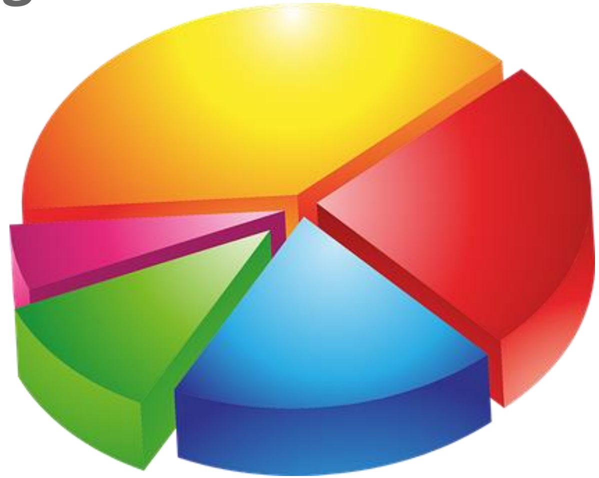

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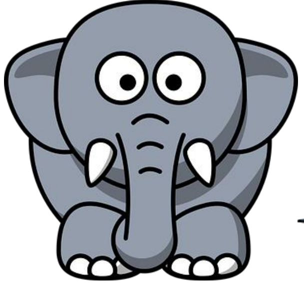

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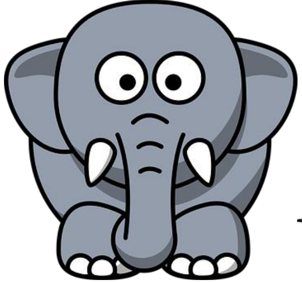


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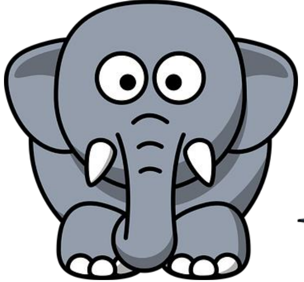
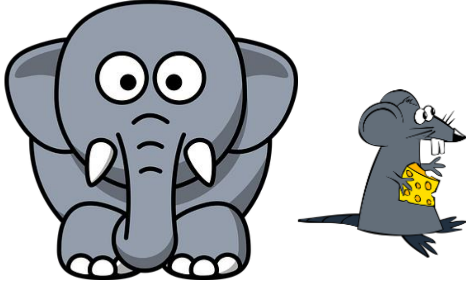
  - What is the expected number of ***siblings*** a random child has?
    - **Choices**: 0, 1, 2, 2.5, 3, 3.33, 4 (you might guess 2?)
  - There are 50*5=250 children with 4 siblings, and 50*1=50 children with 0 siblings.
    - 250/300 * 4 + 50/300 * 0 = 3.3333

Size-Based Sampling




- Let's say there are 100 families. 50 families have five children each, and 50 families only have a single child.
  - What is the expected number of ***siblings*** a random child has?
    - **Choices**: 0, 1, 2, 2.5, 3, 3.33, 4 (you might guess 2?)
  - There are 50*5=250 children with 4 siblings, and 50*1=50 children with 0 siblings.
    - 250/300 * 4 + 50/300 * 0 = 3.3333
  - Actually, it was ambiguous what "random child" meant:

Size-Based Sampling



- UW says the average class size is 28. Do you think that is true, or does it feel that way?
- To simplify, let's say there are 300 students, and each student takes exactly one of three classes.

Class	# Students
1	278
2	10
3	12

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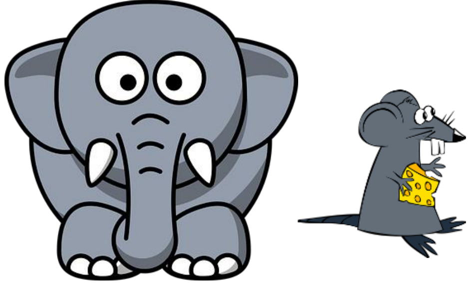

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Average (over each class): $278 \cdot \frac{1}{3} + 10 \cdot \frac{1}{3} + 12 \cdot \frac{1}{3} = 300 \cdot \frac{1}{3} = 100$

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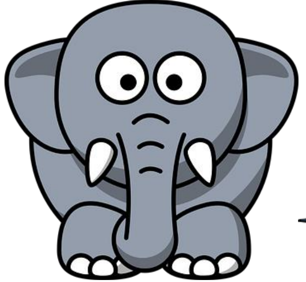

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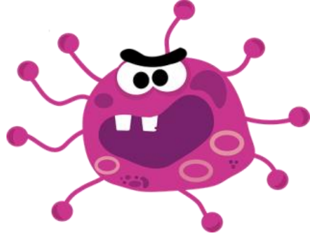
$$\text{Average (over each student): } 278 \cdot \frac{278}{300} + 10 \cdot \frac{10}{300} + 12 \cdot \frac{12}{300} \approx 258.43$$

Random Quote

“Statistics is the grammar of science”

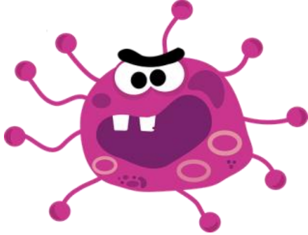
- Karl Pearson

Conditional Probability



- A disease test is 99% accurate, and 0.005% of the population has the disease. If you test positive: the probability you have the disease is:

Conditional Probability



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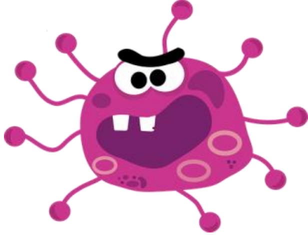
Conditional Probability



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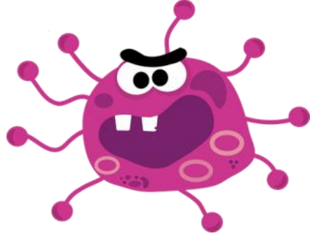

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Much lower than we initially thought! Sometimes non-intuitive...

Conditional Probability

P(Attacked by Alien) = 0.10%

P(Attacked by Alien | AlienShield) = 0.01%

Conditional Probability

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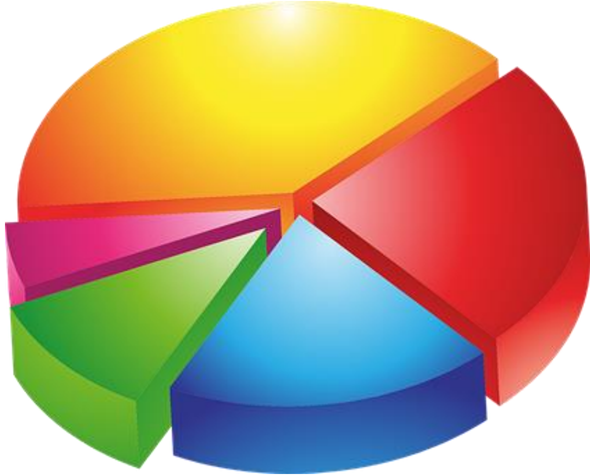
If you are AlienShield, which advertisement do you prefer?

1. (**Relative** Improvement) “AlienShield reduces your chance of getting attacked by an alien 10-fold!”
2. (**Absolute** Improvement) “AlienShield reduces your chance of getting attacked by an alien by 0.09%.”

Poll 7



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Watch for which type of improvement is cited, and consider if the original probability was already low or high.

Conditional Probability

- Suppose there is a carnival game which gives out prizes, and three types of players: children, teenagers, and adults.
- Bob thinks the carnival unfairly gives more prizes to children over the other types of players. Is this true?

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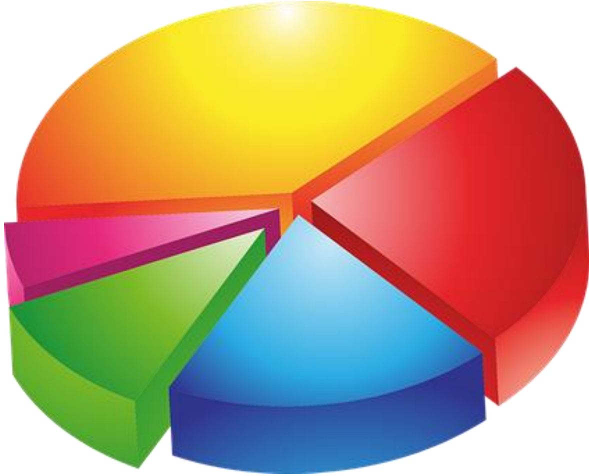
Player Type	% Prizes Won
Children	70%
Teenagers	5%
Adults	25%

Poll 8a

Is this unfair?



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Conditional Probability

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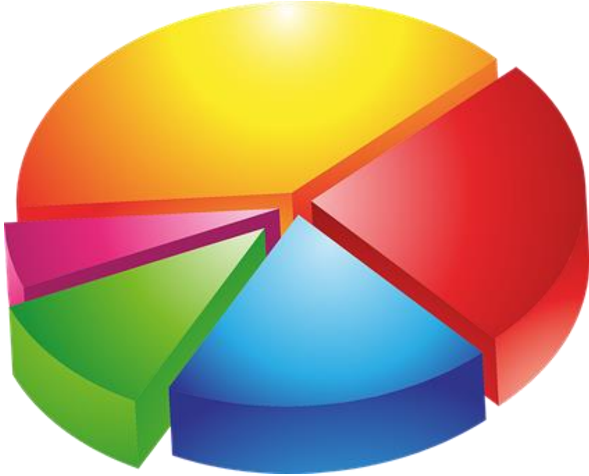
Player Type	% Prizes Won	% Global Population
Children	70%	25%
Teenagers	5%	15%
Adults	25%	60%

Poll 8b

Is this unfair?



Player Type	% Prizes Won	% Global Population
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Adults	25%	60%

Conditional Probability

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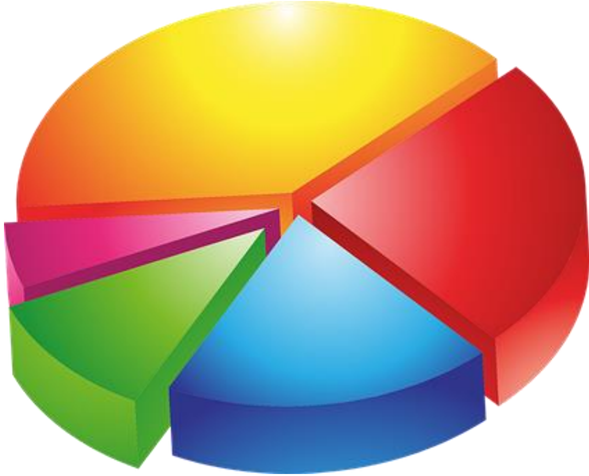
Player Type	% Prizes Won	% Global Population	% Carnival Population
Children	70%	25%	71%
Teenagers	5%	15%	4.5%
Adults	25%	60%	24.5%

Poll 8c

Is this unfair?



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Adults	25%	~~60%~~	24.5%

Looks very fair now!

Conditional Probability

P(child | prize) = 70%

P(child) = 71%

P(teen | prize) = 5%

P(teen) = 4.5%

P(adult | prize) = 25%

P(adult) = 24.5%

Player Type	% Prizes Won	~~% Global Population~~	% Carnival Population
Children	70%	~~25%~~	71%
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Player Type and Prize are (almost) independent!

Conditional Probability

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P(teen | prize) = 5%

P(adult | prize) = 25%

P(child) = 71%

P(teen) = 4.5%

P(adult) = 24.5%

Hypothesis Test: "chi-squared test of independence"

Player Type and Prize are (almost) independent!

Conditional Probability

Statement: "Most people who win a nobel prize went to college."

- P(college | nobel prize) ≈ 1

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There is a big difference between P(A | B) and P(B | A)!!!

Gambler’s Fallacy

- “Play another round of blackjack - you have to win soon! You’ve been losing too much.”

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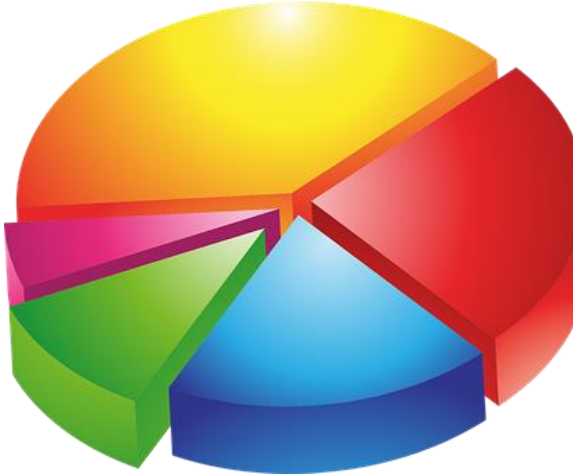
  - “Memorylessness” for Geometric RV.
  - P(win | 100 losses) = P(win | 10 losses) = P(win)

Poll 9

What advice would you give to your friend who has lost 10 consecutive hands of HoldEm and nearly $1000?



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Terrible Advice: “Play another round of blackjack - you have to win soon! You’ve been losing too much.”

Good Advice: “Cut your losses and go home. Quit while you’re ahead”.

Best Advice: “Stop gambling, you idiot.” - A Caring Friend (who understands statistics).

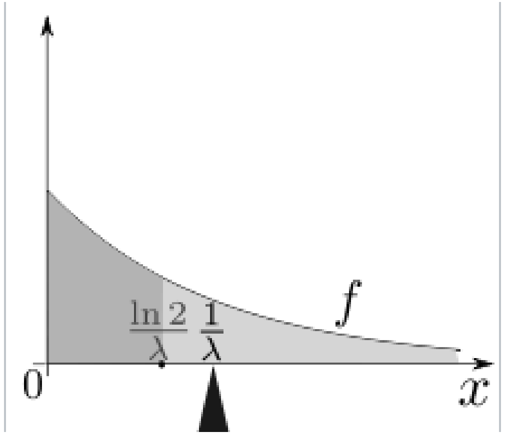
Random Quote

"I guess I think of lotteries as a tax on the mathematically challenged." - Roger Jones

The Well-Chosen Average

- **Mean** (average of all values weighted by probability or density)

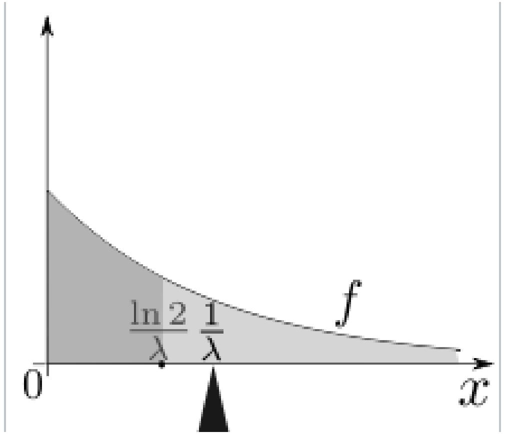
Let $X \sim Exp(\lambda)$. $E[X] = \frac{1}{\lambda}$

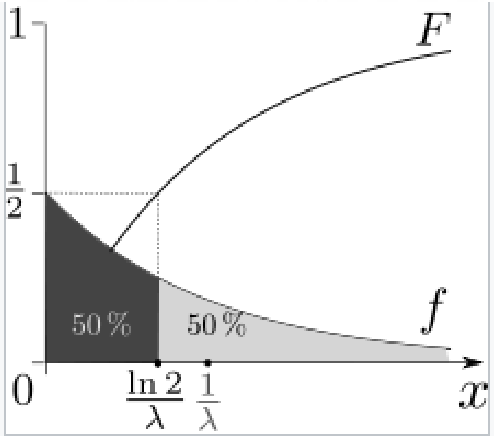


The Well-Chosen Average

- **Mean** (average of all values weighted by probability or density)
- **Median** (the point m where 1/2 values are larger, and 1/2 are smaller)

Let $X \sim Exp(\lambda)$. $E[X] = \frac{1}{\lambda}$ $\text{median}[X] = \frac{\ln 2}{\lambda}$





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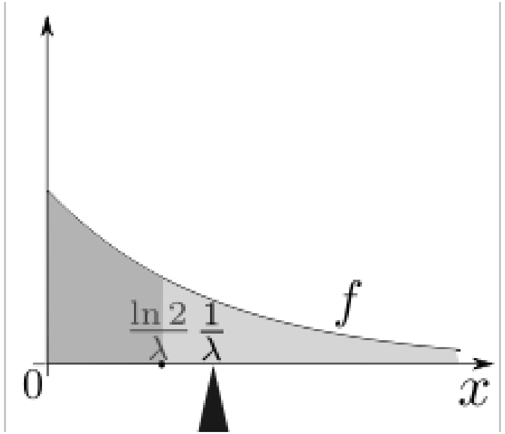

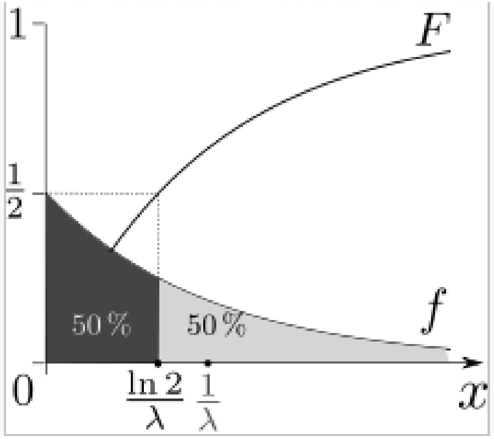

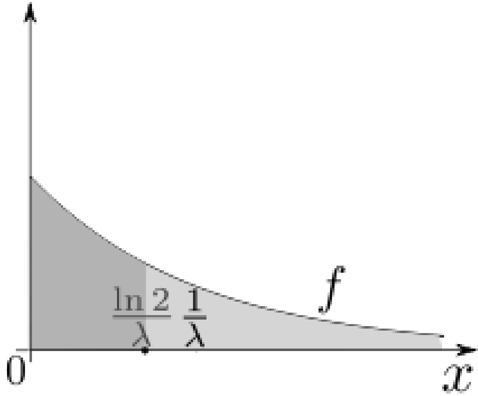

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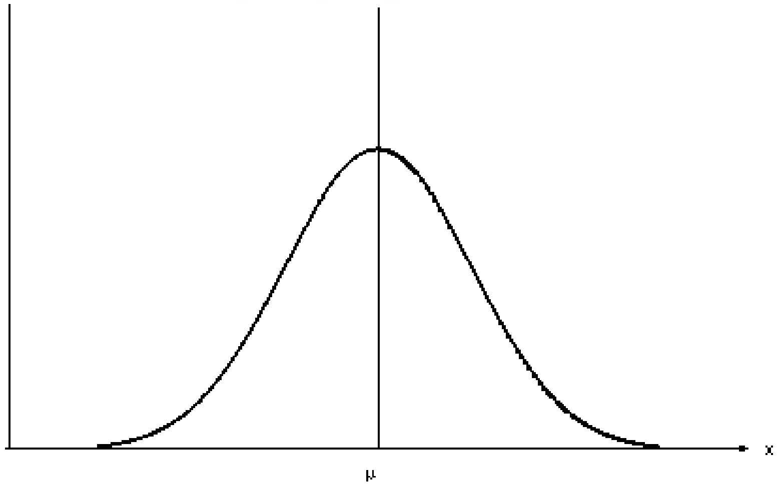
Let $X \sim N(\mu, \sigma^2)$. $E[X] = \mu$ $\text{median}[X] = \mu$ $\text{mode}[X] = \mu$



The Well-Chosen Average

- Are haircuts more expensive in Toronto or Vancouver?

Haircut Prices	Vancouver	Toronto
x_1	$20	$15
x_2	$20	$25
x_3	$22	$25
x_4	$24	$29
x_5	$25	$35
x_6	$28	$45
x_7	$400	$65

Poll 2

Are haircuts more expensive in Toronto or Vancouver?

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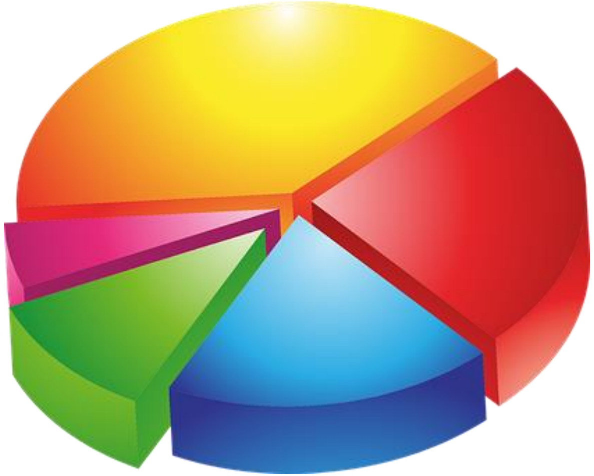

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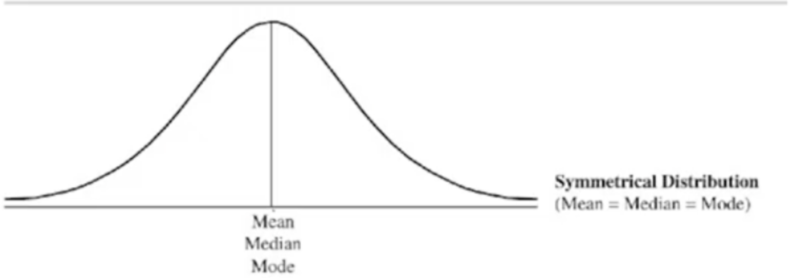
x_7	$400	$65
Mean	$77	$36
Median	$24	$29
Mode	$20	$25

The Well-Chosen Average



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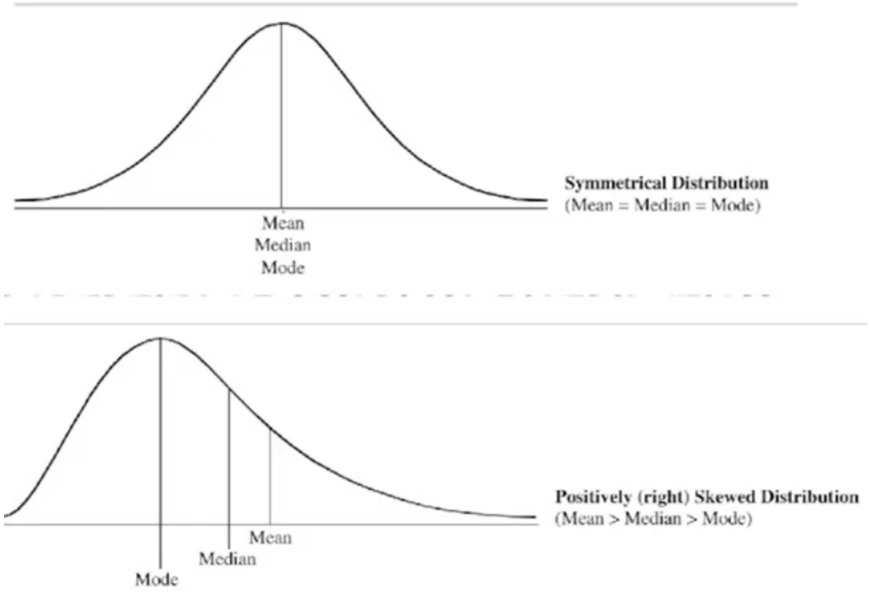

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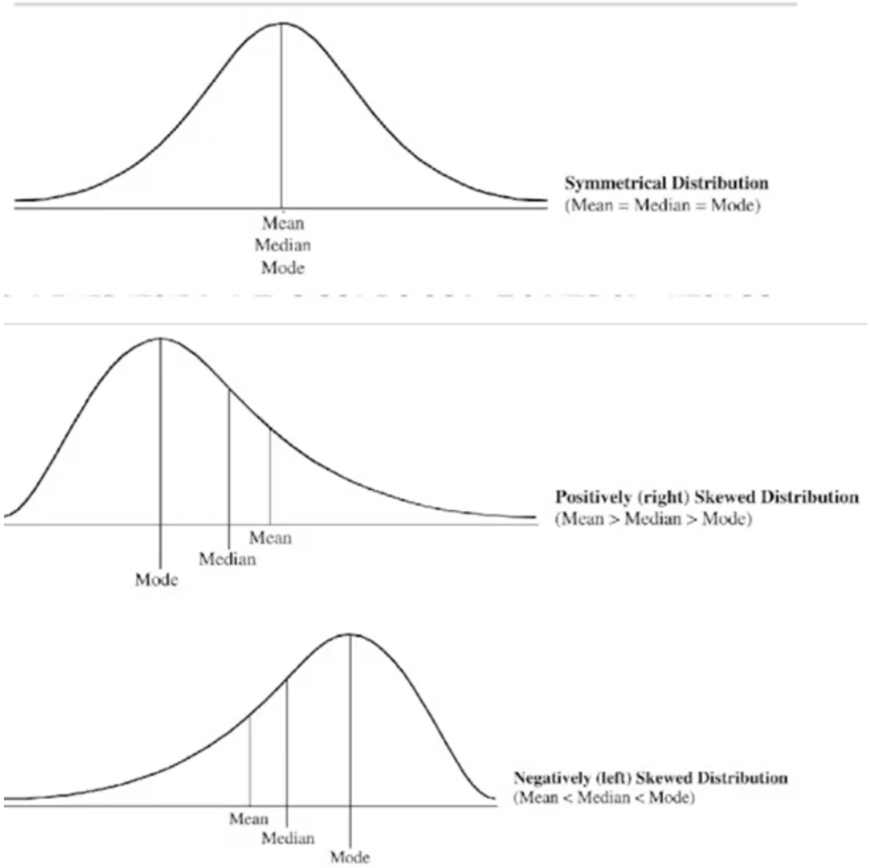

The Well-Chosen Average

- **Mean**: Heavily affected/influenced by outliers. Any extreme value(s) may make this measure terrible.
- **Median**: About half the values are higher, and half are lower than this.
- **Mode**: The most frequently occurring value.

Which is “best”?

The Well-Chosen Average

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- **Median**: About half the values are higher, and half are lower than this.
- **Mode**: The most frequently occurring value.

Which is “best”?

It depends, and it’s good to know all of them for a better idea of the distribution!

Conclusions

1. Determine if the samples are **random** and **representative**.
2. Ask for a confidence interval.
3. Be dubious. Be extremely dubious.
4. Don't make up statistics. You'll get caught.
5. Be wary of p-hacking (and don't do it yourself)!
6. Be careful about seeing patterns where there are none.
7. Correlation does not imply causation.
8. Be careful with interpreting conditional probabilities. Intuition sometimes doesn't work here!
9. Be wary of assuming things are independent that aren't independent.