CSE 312: Foundations of Computing II

Section 8: MLE, MoM, Beta

1. 312 Grades

Suppose Professor Alex loses everyones grades for 312 and decides to make it up by assigning grades randomly according to the following probability distribution, and hoping the n students wont notice: give an A with probability 0.5, a B with probability θ, a C with probability 2θ, and an F with probability $0.5 - 3\theta$. Each student is assigned a grade independently. Let x_A be the number of people who received an A, x_B the number of people who received a B, etc, where $x_A + x_B + x_C + x_F = n$. Find the MLE for θ.

2. A Red Poisson

Suppose that $x_1, \ldots, x_n$ are i.i.d. samples from a Poisson(θ) random variable, where θ is unknown. Find the MLE of θ.

3. Independent Shreds, You Say?

You are given 100 independent samples $x_1, x_2, \ldots, x_{100}$ from Bernoulli(θ), where θ is unknown. (Each sample is either a 0 or a 1). These 100 samples sum to 30. You would like to estimate the distribution's parameter θ. Give all answers to 3 significant digits. What is the maximum likelihood estimator $\hat{\theta}$ of θ?

4. Y Me?

Let $y_1, y_2, ...y_n$ be i.i.d. samples of a random variable with density function

$$f_Y(y|\theta) = \frac{1}{2\theta} \exp(-\frac{|y|}{\theta})$$

.

Find the MLE for θ in terms of $|y_i|$ and n.

5. Pareto

The Pareto distribution was discovered by Vilfredo Pareto and is used in a wide array of fields but particularly social sciences and economics. It is a density function with a slowly decaying tail, for example it can describe the wealth distribution (a small group at the top holds most of the wealth). The PDF is given by:

$$f_X(x; m, \alpha) = \frac{\alpha m^\alpha}{x^{\alpha+1}}$$

where $x \geq m$ and real $\alpha, m > 0$. m describes the minimum value that X takes on (scale) and α is the shape. So the range of X is $\Omega_X = [m, \infty)$. Assume that m is given and that $x_1, x_2, \ldots, x_n$ are i.i.d. samples from the Pareto distribution. Find the MLE estimation of α.

6. MOM Practice

Let $X_1, ..., X_n$ be a random sample from the distribution with PDF $f_X(x \mid \theta) = (\theta^2 + \theta)x^{\theta-1}(1 - x)$ for $0 < x < 1$ and $\theta > 0$. What is the MOM estimator for θ?

7. Laplace

Suppose $x_1, \ldots, x_{2n}$ are iid realizations from the Laplace density (double exponential density)

$$f_X(x \mid \theta) = \frac{1}{2} e^{-|x-\theta|}$$

Find the MLE for θ. For this problem, you need not verify that the MLE is indeed a maximizer. You may find the **sign** function useful:

$$\text{sgn}(x) = \begin{cases} +1, & x > 0 \\ -1, & x < 0 \end{cases}$$

(in our case undefined at 0)

8. Beta

(a) Suppose you have a coin where you have no prior belief on its true probability of heads p. How can you model this belief as a Beta distribution?

(b) Suppose you have a coin which you believe is fair, with strength α. That is, pretend youve seen α heads and α tails. How can you model this belief as a Beta distribution?

(c) Now suppose you take the coin from the previous part and flip it 10 times. You see 8 heads and 2 tails. How can you model your posterior belief of the coins probability of heads?