[**Tags:** Joint PDFs, Marginal PDFs, Expectation]

1. Suppose we have a joint density as follows:

$$f_{X,Y}(x,y) = \begin{cases} cx^2y^4, & x^2 - 1 \le y \le 1 - x^2 \\ 0, & \text{otherwise} \end{cases}$$

   a. Sketch the joint range $\Omega_{X,Y}$, and determine the marginal ranges Ω_X, Ω_Y from the picture.
   b. Write an expression for the value c that makes $f_{X,Y}$ a valid joint density function.
   c. Write an expression for $f_X(x)$.
   d. Write an expression for $f_Y(y)$.
   e. Write an expression for $E\left[\frac{1}{X^2+Y^2}\right]$.

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2. Suppose (X, Y, Z) are jointly distributed with density function:

$$f_{X,Y,Z}(x,y,z) = \begin{cases} ce^{-13x}e^{-13y}, & x, y > 0 \text{ and } 0 < z < 47 \\ 0, & \text{otherwise} \end{cases}$$

The joint range is a nice "rectangle" (rectangular prism with infinitely large base technically), which we unfortunately can't visualize.

   a. Set up an appropriate triple integral with the order $dxdydz$ for the value of c, which turns out to be $13^2/47$.
   b. Compute $f_X(x)$ using WolframAlpha after setting up the limits of integration (you should integrate over ALL other variables). And note $f_Y(y)$ is identical by symmetry. Actually, X, Y have the same distribution from our zoo – which is it?
   c. Compute $f_Z(z)$ using WolframAlpha. Actually, Z also has a distribution from our zoo – which is it?
   d. Are X, Y, Z mutually independent? (This means, $f_{X,Y,Z}(x,y,z) = f_X(x)f_Y(y)f_Z(z)$, and this also must hold true for each pair's marginal joint distribution, but let's not worry about that now.) Note that the joint range must also be the Cartesian product of the three marginal ranges.
   e. Write an expression for $E\left[\log\left(\frac{1}{Z^{X+Y}}\right)\right]$.