CSE 312: Foundations of Computing II
Instructor: Alex Tsun
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Lecture Topics: 5.4 Covariance and Correlation

[**Tags:** Covariance]

1. The covariance matrix of a random vector $\boldsymbol{Z} = (Z_1, Z_2, \dots, Z_n)$ is defined to the $n \times n$ matrix Σ such that $\Sigma_{ij} = Cov(X_i, X_j)$. Don't be too intimidated - it's just a way to store all the information we need – we won't be doing any linear algebra with it! The examples will help ☺.

$$\Sigma = \begin{bmatrix} Cov(Z_1, Z_1) = Var(Z_1) & Cov(Z_1, Z_2) & \dots & Cov(Z_1, Z_n) \\ Cov(Z_2, Z_1) & Cov(Z_2, Z_2) = Var(Z_2) & \dots & Cov(Z_2, Z_n) \\ \vdots & \ddots & \ddots & \vdots \\ Cov(Z_n, Z_1) & \dots & \dots & Cov(Z_n, Z_n) = Var(Z_n) \end{bmatrix}$$

   a. Let X_1, X_2, X_3, X_4 be iid (independent and identically distributed) random variables with mean μ and variance σ^2. Let $\boldsymbol{X} = (X_1, X_2, X_3, X_4)$ be a random vector with the rvs X_i as its components. What is the (4×4) covariance matrix of $\boldsymbol{X}$?
   b. Define $\boldsymbol{Y} = (Y_1, Y_2, Y_3)$ as follows.
      - $Y_1 = X_1 + X_2$
      - $Y_2 = X_2 + X_3$
      - $Y_3 = X_3 + X_4$.

      What is the (3×3) covariance matrix of $\boldsymbol{Y}$?

[**Tags:** Similar to PSet4 Q3, Covariance]

2. Suppose we throw 12 balls independently and uniformly into 7 bins. For $i = 1, \dots, 7$, let X_i be the indicator/Bernoulli rv of whether bin i is empty. Let $\boldsymbol{X} = (X_1, \dots, X_7)$ be the random vector of indicators.
   a. What is the covariance matrix of $\boldsymbol{X}$?
   b. Let $Y = \sum_{i=1}^{7} X_i$ be the number of empty bins. What is $Var(Y)$?