CSE 312

Foundations of Computing II

Lecture 1: Introduction & Counting

https://cs.washington.edu/312

Instructor

Rachel Lin [she/her]

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Specialty: **Cryptography (uses a LOT of probability)**

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A Team of fantastic TAs

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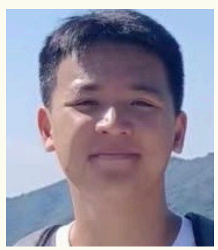

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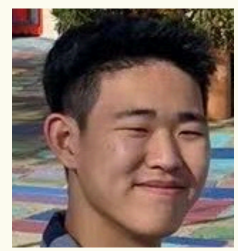

Andew Mingwei Zhang

Jolie Zhou

See https://courses.cs.washington.edu/courses/cse312/23au/staff.html to learn more about their backgrounds and interests!

Lectures and Sections

- **Lectures MWF (SAV 260)**
  - 1:30-2:20pm
  - Classes will be in person
  - Lectures are recorded
    - Panopto automatic recording and available on Canvas
  - Annotated slides also uploaded.
- **Sections Thu (starts this week)**
  - Not recorded
  - Will prepare you for problem sets!

Interaction in Class

- **Poll Everywhere**
  - We will sometimes use Poll Everywhere during class
  - You sign up directly

Go to https://www.polleverywhere.com/login and login using YOURNETID@uw.edu

We use pollev.com/rachel312

- **Online Chat in Class**
  - Ask questions related to the lecture, e.g., what does XX notation mean? Can some one remind me what is the product rule?
  - Chime in to answer questions from your fellow classmates.
  - Express your insights, thoughts, ideas. Say hello.
  - **We have one TA coming into each lecture to answer questions.**

Today: Ed chat Channel: in-lecture-discussion-0927

Questions and Discussions

- **Office hours throughout the week (starting this Thursday)**
  - See https://courses.cs.washington.edu/courses/cse312/23au/staff.html
- **Ed Discussion**
  - You should have received an invitation (synchronized with the class roaster)
  - Material (resources tab)
  - Announcements (discussion tab)
  - Discussion (discussion tab)

Use Ed discussion forum as much as possible. You can make private posts that only the staff can view! Email instructor for personal issues.

Engagement

- **"Concept checks" after each lecture 5-8 %**
  - Must be done (on Gradescope) before the next lecture by 1:00 pm.
  - Simple questions to reinforce concepts taught in each class
  - Keep you engaged throughout the week, so that homework becomes less of a hurdle
- **9 Problem Sets (Gradescope) 45-50 %**
  - Solved individually. Discussion with others allowed but separate solutions
  - Homework released every week, except Week 6 (midterm), and Week 11. See homework schedule https://courses.cs.washington.edu/courses/cse312/22au/index.html
  - Generally due Wednesdays, but Fridays after Thanksgiving
  - First problem set posted later today
- **Midterm 15-20 %**
  - In class on **Wednesday, Nov 2 (Week 6)**
- **Final Exam 30-35 %**
  - **Monday, December 12** at **2:30-4:20 pm** in this room (as in UW Autumn Quarter Exam Schedule)

For more details see

Course Webpage **https://cs.washington.edu/312**

e.g., late policies on concept checks and homeworks

Foundations of Computing II

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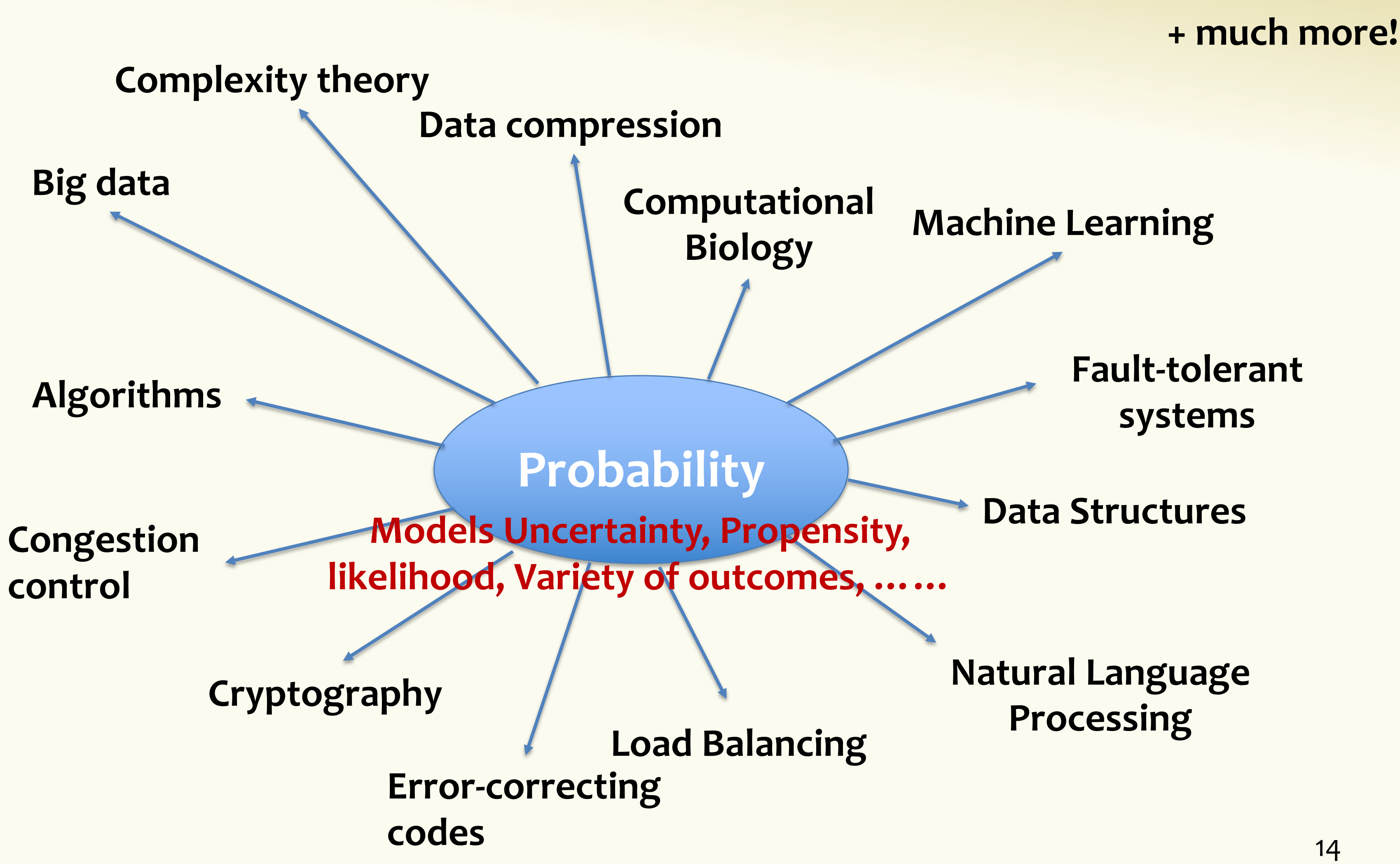
Introduction to Counting, Probability & Statistics

for <u>computer scientists</u>

<u>What</u> is probability??

<u>Why</u> probability?!


+ much more!
Complexity theory
Data compression
Big data
Computational Biology
Machine Learning
Algorithms
Fault-tolerant systems
Probability
Data Structures
Congestion control
Models Uncertainty, Propensity, likelihood, Variety of outcomes, ……
Cryptography
Natural Language Processing
Load Balancing
Error-correcting codes

Content

- Counting (basis of discrete probability)
  - Counting, Permutation, Combination, inclusion-exclusion, Pigeonhole Principle
- What is probability
  - Probability space, events, basic properties of probabilities, conditional probability, independence, expectation, variance
- Properties of probability
  - Various inequalities, Zoo of discrete random variables, Concentration, Tail bounds
- Continuous Probability
  - Probability Density Functions, Cumulative Density Functions, Uniform, Exponential, Normal distributions, Central Limit Theorem, Estimation
- Applications
  - A sample of randomized algorithms, differential privacy, learning …

Today: A fast introduction to counting so you will have enough to work on in section tomorrow…

We are interested in counting the number of objects with a certain given property.

"How many ways are there to assign 7 TAs to 5 sections, such that each section is assigned to two TAs, and no TA is assigned to more than two sections?"

"How many positive integer solutions (x, y, z) *does the equation* $x^3 + y^3 = z^3$ *have?"*

Generally: Question boils down to computing cardinality $|S|$ of some given set S.

(Discrete) Probability and Counting are Twin Brothers

"What is the probability that a random student from CSE312 has black hair?"

$$= \frac{\#\ students\ with\ black\ hair}{\#students}$$

Today – Two basic rules

- Sum rule
- Product rule
- Permutation and Combination

Sum Rule

If you can choose from

- EITHER one of n options,
- OR one of m options with NO overlap with the previous n

then the number of possible outcomes of the experiment is

$$n + m$$

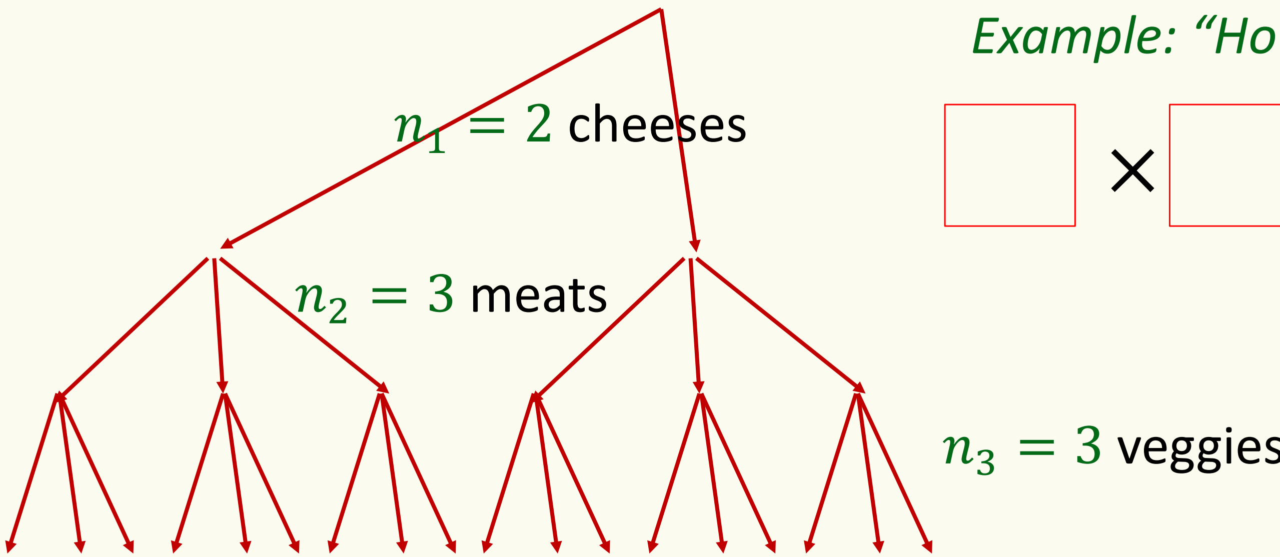
Counting "lunches"

If a lunch order consists of **either** one of 6 soups **or** one of 9 salads, how many different lunch orders are possible?

Product Rule: In a sequential process, there are

- n_1 choices for the first step,
- n_2 choices for the second step (given the first choice), …, and
- n_m choices for the m^{th} step (given the previous choices),

then the total number of outcomes is $n_1 \times n_2 \times \cdots \times n_m$



Example: "How many subs?"

$\square \times \square \times \square = \square$

Product rule examples – Strings

How many strings of length 5 over the alphabet $\{A, B, C, \dots, Z\}$ are there?

- E.g., AZURE, BINGO, TANGO, STEVE, SARAH, …

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How many binary strings of length n over the alphabet $\{0,1\}$?

- E.g., $0 \cdots 0, 1 \cdots 1, 0 \cdots 01, \dots$

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Product rule example – Cartesian Product

Definition. The **cartesian product** of two sets S, T is

$$S \times T = \{(a, b): a \in S, b \in T\}$$

Called a 2-sequence
Order matters! $(a, b) \neq (b, a)$

$$|S \times T| = \square \times \square$$

$$|A_1 \times A_2 \times \cdots \times A_n| = \square \times \square \times \square \times \cdots \times \square$$

Product rule example – Power set

Definition. The **power set** of S is the set of all subsets of S,

$$\{X : X \subseteq S\}.$$

Notations: $\mathcal{P}(S)$ or simply 2^S (which we will use).

Example. $2^{\{\bigstar, \spadesuit\}} = \{\emptyset, \{\bigstar\}, \{\spadesuit\}, \{\bigstar, \spadesuit\}\}$

$2^{\emptyset} = \{\emptyset\}$

…

How many different subsets of S are there if $|S| = n$?

Product rule example – Power set

set $S = \{e_1, e_2, e_3, \cdots, e_n\}$

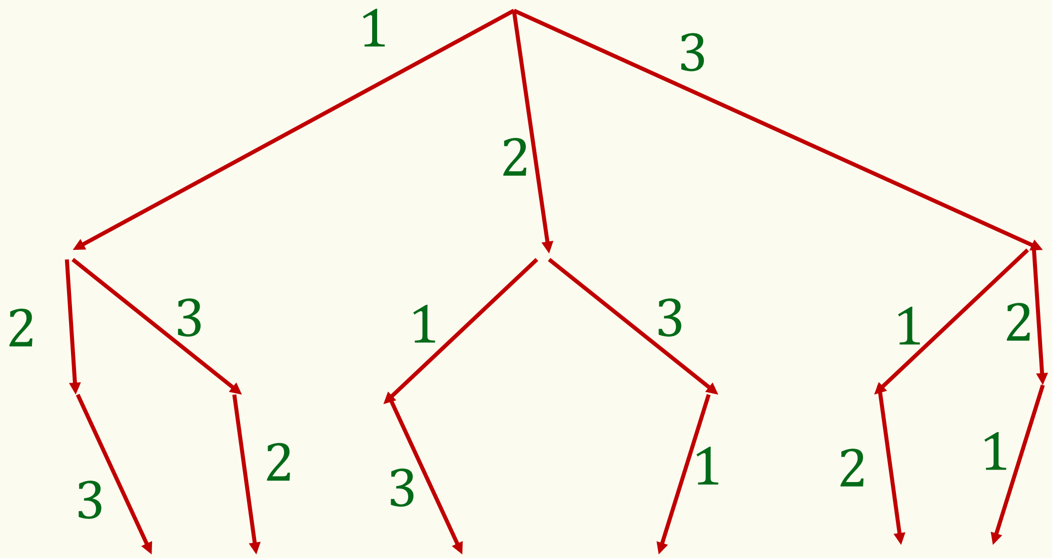
subset $X = \{ \qquad \}$

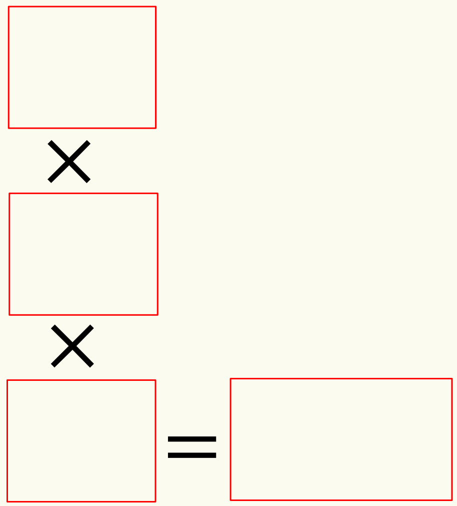
$\square \times \square \times \square \times \cdots \times \square = \square$

Proposition. $|2^S| = 2^{|S|}$

Note: Sequential process for product rule works even if the sets of options are different at each point

"How many sequences in $\{1,2,3\}^3$ *with no repeating elements?"*





Factorial

"How many ways to order elements in S, where $|S| = n$?" **Permutations**

Answer = $n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$

Definition. The **factorial function** is

$$n! = n \times (n-1) \times \cdots \times 2 \times 1$$

Note: $0! = 1$

Theorem. (Stirling's approximation)

$$\underbrace{\sqrt{2\pi}}_{= 2.5066} \cdot n^{n+\frac{1}{2}} \cdot e^{-n} \leq n! \leq \underbrace{e}_{= 2.7183} \cdot n^{n+\frac{1}{2}} \cdot e^{-n}$$

Huge: Grows exponentially in n

Nice use of sum rule: Counting using complements

"How many sequences in $\{1,2,3\}^3$ *have repeating elements?"* m

"# of sequences in $\{1,2,3\}^3$ *with no repeating elements"* $n =$

"# of sequences in $\{1,2,3\}^3$ $3^3 = 27$ $= m + n$ *by the sum rule*

All sequences

$m = 27 - n =$

Distinct Letters

"How many sequences of 5 ***distinct*** *alphabet letters from* $\{A, B, \ldots, Z\}$*?"*

E.g., AZURE, BINGO, TANGO. But not: STEVE, SARAH

Answer: $26\times25\times24\times23\times22 = 7893600$

In general

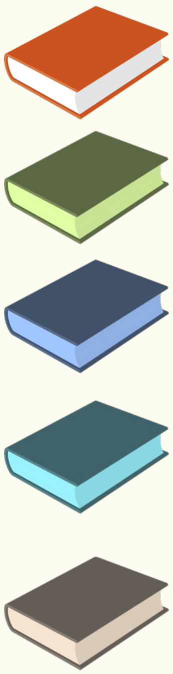
Aka: k-permutations

Fact. # of k-element sequences of distinct symbols from an n-element set is

$$P(n,k) = n \times (n-1) \times \cdots \times (n-k+1) = \frac{n!}{(n-k)!}$$

Product rule – One more example

5 books



"How many ways are there to distribute 5 books among Alice, Bob, and Charlie?"

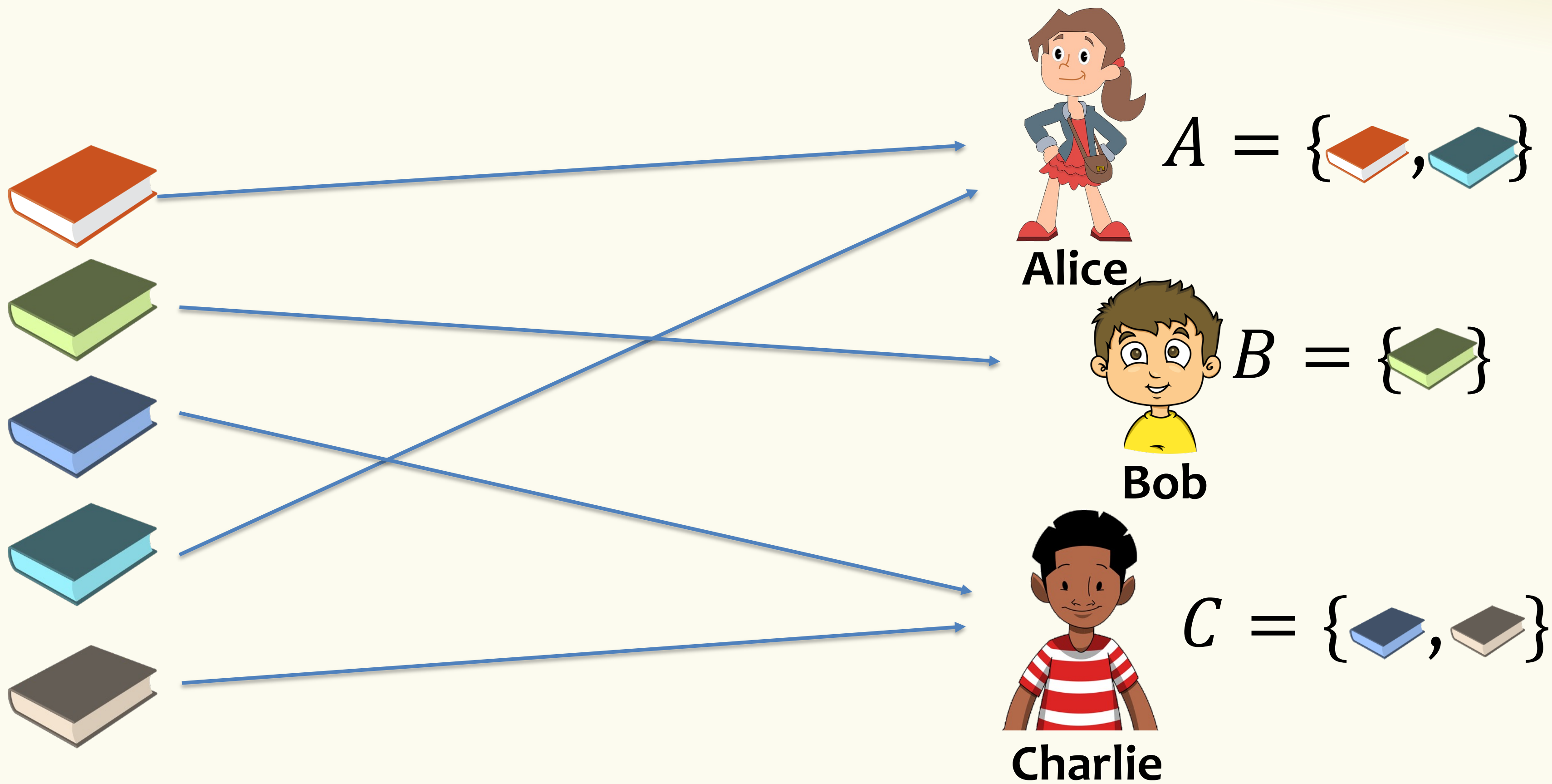
Every book to one person, everyone gets ≥ 0 books.

Alice

Bob

Charlie

Example Book Assignment



Book assignment – Modeling

Correct?

Poll:

A. Correct
B. Overcount
C. Undercount
D. No idea

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$2^5 = 32$ options

$\times$

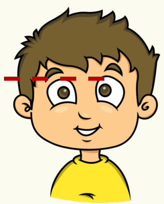
$2^5 = 32$ options

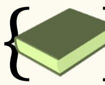
$\times$

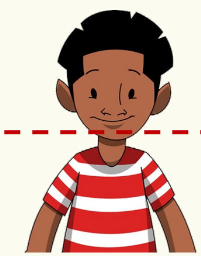
$2^5 = 32$ options

$= 32^3$ assignments

$A = \{$  $\}$

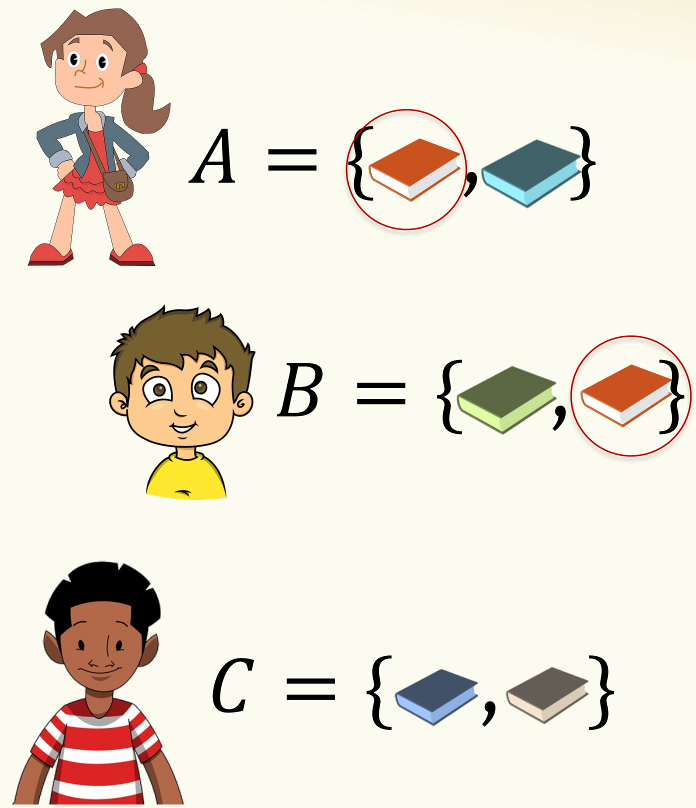


$B = \{$  $\}$



$C = \{$  $\}$

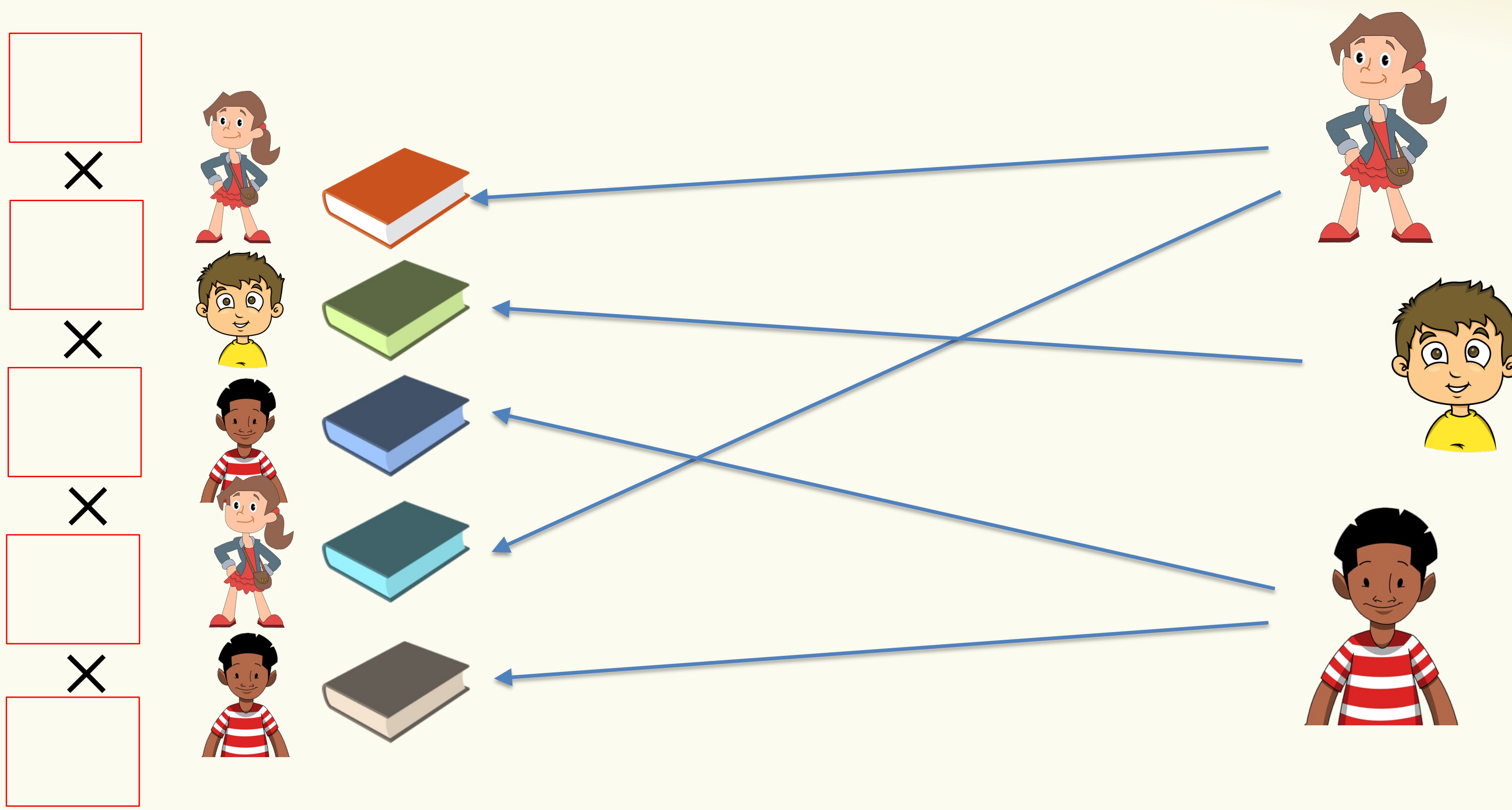
Problem – Overcounting



Problem: We are counting some <u>invalid</u> assignments!!!
$\longrightarrow$ **<u>overcounting</u>**!

What went wrong in the sequential process?
After assigning A to Alice,
B is no longer a valid option for Bob

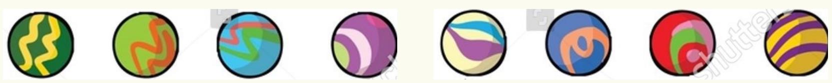
Book assignments – A Clever Approach



Lesson: Representation of what we are counting is very important!

Tip: Use different methods to double check yourself
Think about counter examples to your own solution.

- **Permutations and Combinations**



Number of Subsets

"How many size-5 ***subsets*** *of* $\{A, B, \dots, Z\}$*?"*

E.g., {A,Z,U,R,E}, {B,I,N,G,O}, {T,A,N,G,O}. But not: {S,T,E,V}, {S,A,R,H},…

Difference from k-permutations: NO ORDER

Different sequences: TANGO, OGNAT, ATNGO, NATGO, ONATG …

Same set: {T,A,N,G,O}, {O,G,N,A,T}, {A,T,N,G,O}, {N,A,T,G,O}, {O,N,A,T,G}… …

Number of Subsets – Idea

Consider a sequential process:

1. Choose a subset $S \subseteq \{A, B, \dots, Z\}$ of size $|S| = 5$
   e.g. $S = \{A, G, N, O, T\}$
2. Choose a permutation of letters in S
   e.g., *TANGO, AGNOT, NAGOT, GOTAN, GOATN, NGOAT, …*

$$???$$
$$\times$$
$$5!$$
$$=$$
$$\frac{26!}{21!}$$

Outcome: A sequence of 5 distinct letters from $\{A, B, \dots, Z\}$

$$??? = \frac{26!}{21!\,5!} = 65780$$

k-combination

Fact. The number of subsets of size k of a set of size n is

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

a.k.a. Binomial coefficient (verbalized as "n choose k")

Notation: $\binom{S}{k}$ = set of all k-element subsets of S. $\left|\binom{S}{k}\right| = \binom{|S|}{k}$

[also called **combinations**]

Quick Summary

- k-sequences: How many length k sequences over alphabet of size n?
  - Product rule ➔ n^k
- k-permutations: How many length k sequences over alphabet of size n, without repetition?
  - Permutation ➔ $\frac{n!}{(n-k)!}$
- k-combinations: How many size k subsets of a set of size n (without repetition and without order)?
  - Combination ➔ $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

The first concept check is out and due at 1:00pm before Friday's lecture

The concept checks are meant to help you immediately reinforce what is learned.

Students from previous quarters have found them really useful!