Find a group of 3-5 people to sit with

This is to ensure that we get through all the groups in time when working on problems as groups :)

SECTION 2

-------More Counting & Probability------

LOGISTICS

HW 1 due yesterday
(Late deadline Friday(1/12 @ 11:59pm)

Hw 2 is out
(due Wednesday(1/17 @ 11:59pm)

Office Hours
(times/locations listed on the website)

Homework

- Submissions
  - LaTeX (highly encouraged)
    - overleaf.com
    - template and LaTeX guide posted on course website!
  - Word Editor that supports mathematical equations
  - Handwritten neatly and scanned
- Homework will typically be due on Wednesdays at 11:59pm on Gradescope
- Each assignment can be submitted a max of **48 hours** late
- You have **6 late days total** to use throughout the quarter
  - Anything beyond that will result in a deduction on further late assignments

Content Review

NEW TOPICS!

Binomial Theorem

Inclusion Exclusion

Pigeonhole Principle

Stars and Bars

Probability Spaces and Uniform Probability

Fun Counting Application:

Binomial Theorem

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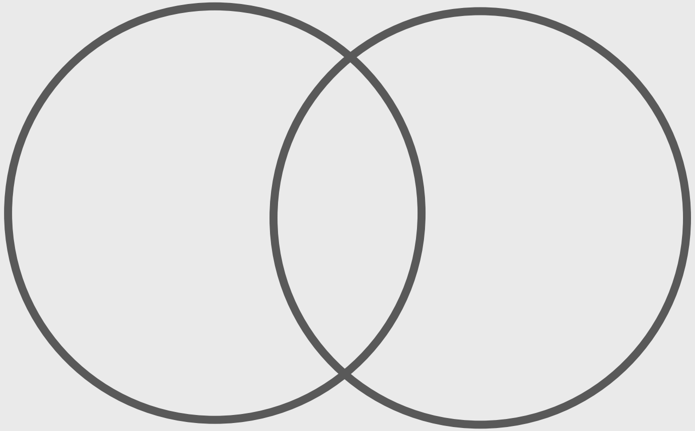
Is this identical to ${}_nC_{n-k}$?

Yes! Choosing a set of k out of n things is the same as choosing a set of n - k things to not include

Another counting rule:

inclusion-exclusion

$|A \cup B|$ *isn't as simple as* $|A|+|B|$



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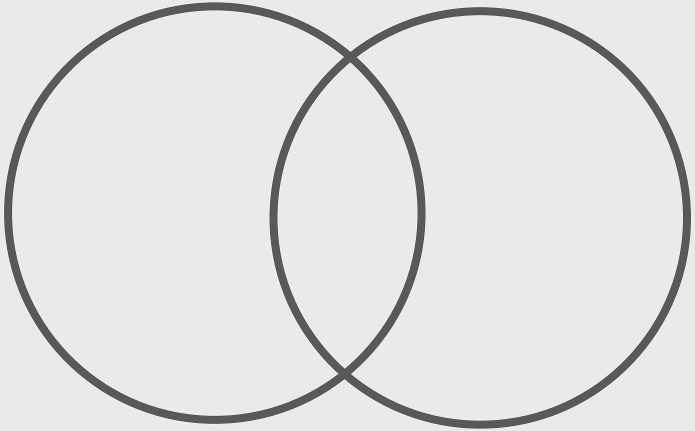
$|A \cup B|$ *is* $|A|+|B|-|A \cap B|$



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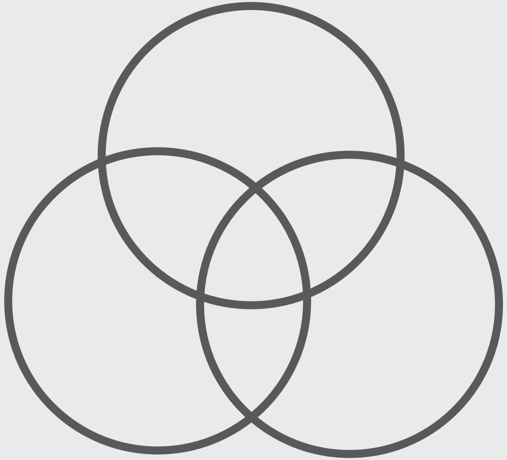

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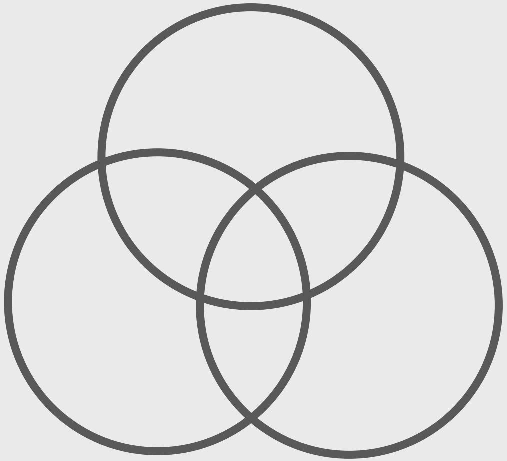
$|A \cup B \cup C|$ *is*

singles - doubles + triples

$|A|+|B|+|C|-|A \cap B|-|A \cap C|-|B \cap C|+|A \cap B \cap C|$



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Pigeonhole Principle

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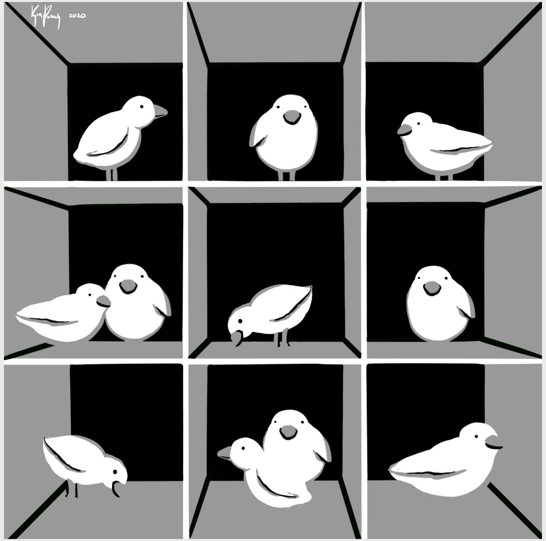
If there are **n** pigeons with not enough holes for them to stay in (**k** to be exact), what can we say about at least how many pigeons at least one hole will hold?



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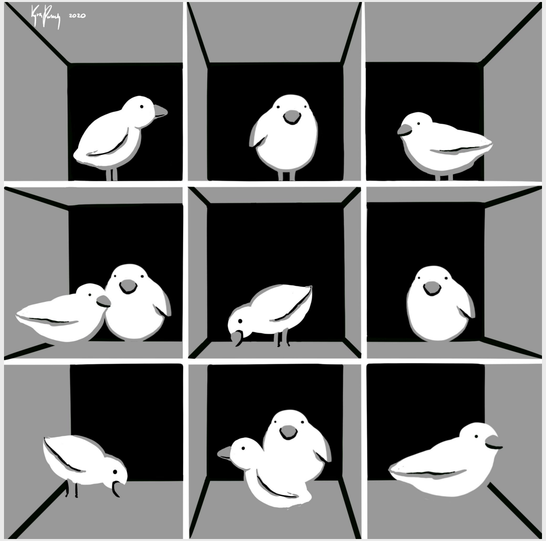
ceil(n / k)



Another counting rule:

STARS AND BARS

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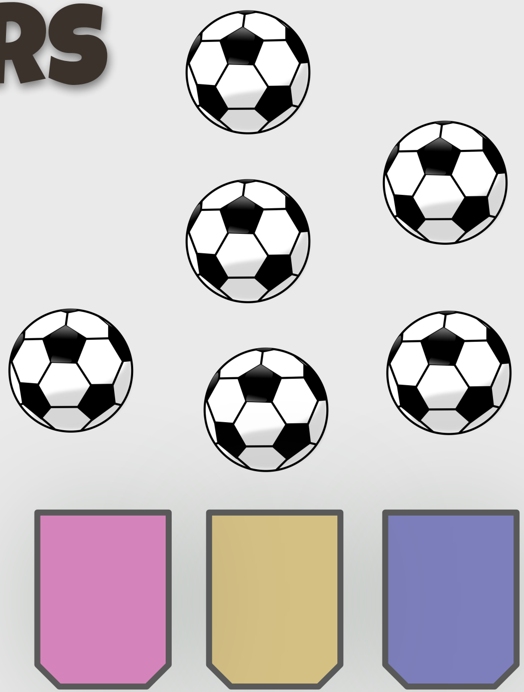
How many ways can you distribute **n** indistinguishable balls into **k** distinguishable bins?



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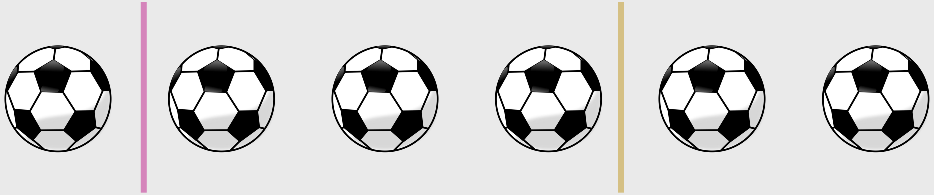
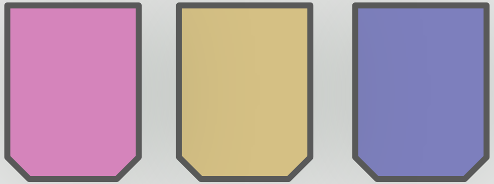
STARS AND BARS

How many ways can you distribute **n** indistinguishable balls into **k** distinguishable bins?

*Arrange **n** balls and **k - 1** dividers*





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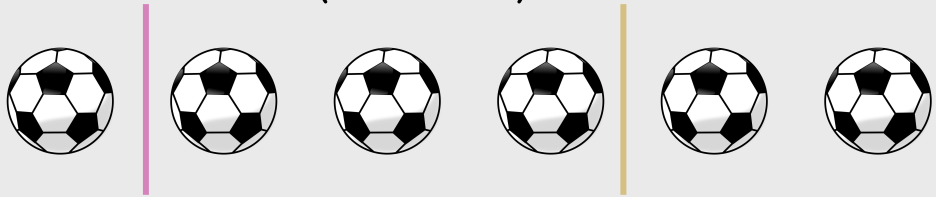
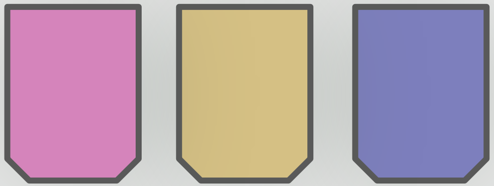
$$_{(n + k - 1)}C_n$$





PROBABILITY!

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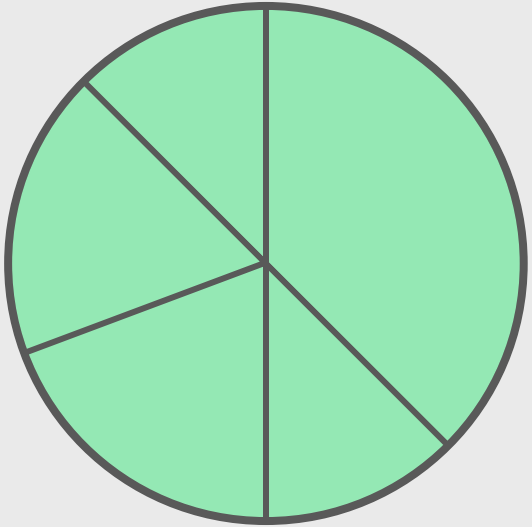
- **Sample Space:** The set of all possible outcomes of an experiment, denoted Ω or S
- **Event:** Some subset of the sample space, usually a capital letter such as $E \subseteq \Omega$
- **Union:** The union of two events E and F is denoted $E \cup F$
- **Intersection:** The intersection of two events E and F is denoted $E \cap F$ or EF
- **Mutually Exclusive:** Events E and F are mutually exclusive iff $E \cap F = \emptyset$
- **Complement:** The complement of an event E is denoted E^C or $\bar{E}$ or $\neg E$, and is equal to $\Omega \setminus \mathrm{E}$
- **DeMorgan's Laws:** $(E \cup F)^C = E^C \cap F^C$ and $(E \cap F)^C = E^C \cup F^C$

PROBABILITY!



Sample Space

Each probability is between 0 and 1 inclusive

Probabilities add to 1

If events are *mutually exclusive*,
$P(A \cup B \cup C) = P(A) + P(B) + P(C)$
because there are no intersections

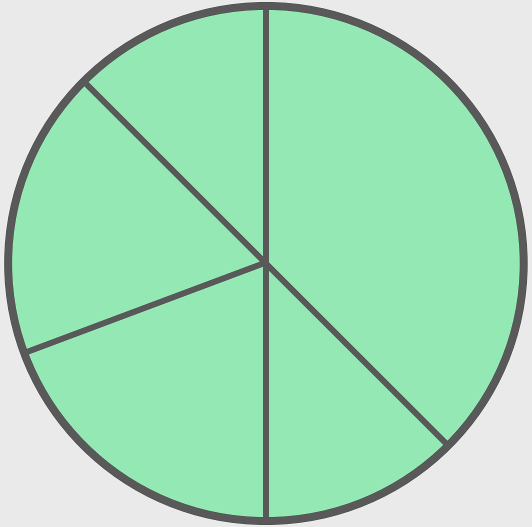
PROBABILITY!

- **Axioms of Probability**
  - **Non-negativity:** For any event E, $\mathbb{P}(E) \geq 0$
  - **Normalization:** $\mathbb{P}(\Omega) = 1$
  - **Additivity:** If E and F are mutually exclusive events, then $\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F)$

- **Corollaries of these axioms**
  - **Complementation**: $\mathbb{P}(E) + \mathbb{P}(E^C) = 1$
  - **Monotonicity**: If $E \subseteq F$, $\mathbb{P}(E) \leq \mathbb{P}(F)$
  - **Inclusion-Exclusion**: $\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$

- **Equally Likely Outcomes**: If every outcome in a finite sample space Ω is equally likely, and E is an event, then $\mathbb{P}(E) = \frac{|E|}{|\Omega|}$

PROBABILITY!



Sample Space

An event is a subset of the sample space

$E \subseteq \Omega$

If each outcome in the sample space is *equally likely*, the probability of an event is

`P(E) = |E| / |Ω|`

If the union of a set of mutually exclusive events is equal to the sample sets, those events *partition* the sample space