CSE 321: Discrete Structures **Winter 2000**

Assignment #6

Due in class: Wednesday, February 23

1. Use the fact that $\binom{p}{i}$ is an integer together with a result from Assignment #5 to show that for all primes p and all integers i such that $0 < i < p$,

$$\binom{p}{i} \bmod p = 0.$$

2. Use the previous result, the binomial theorem, and induction on x to prove that $\forall x \in \mathbb{N}$ and all primes p,

$$x^p \bmod p = x \bmod p.$$

3. Use the previous result and induction on k to prove that $\forall x, k \in \mathbb{N}$ and all primes p,

$$x^{k(p-1)+1} \bmod p = x \bmod p.$$

4. Use the previous result to show that $\forall x, K \in \mathbb{N}$ and for all primes p and q,

$$x^{K(p-1)(q-1)+1} \bmod p = x \bmod p \quad \text{and} \quad x^{K(p-1)(q-1)+1} \bmod q = x \bmod q.$$

5. Use the previous result together with a result from Assignment #5 to show that $\forall x, K \in \mathbb{N}$ and for all primes p and q with $p \neq q$,

$$x^{K(p-1)(q-1)+1} \bmod (pq) = x \bmod (pq).$$