4. Let a and b be positive integers and $g = \gcd(a, b)$.

   Prove or disprove the assertion that there are no positive integers $c < g$ for which there exist integers x and y such that $c = ax + by$.

This is true!

It will be proven by contradiction.

Suppose that there is some positive integer $c < g$ for which there exist integers x and y such that $c = ax + by$.

Since g is a common divisor of a and b, $g \mid a$ and $g \mid b$. Thus, $g \mid ax$ and $g \mid bx$, so $g \mid (ax + by)$ and hence $g \mid c$.

[The conclusion that $g \mid c$ can also be reached by appealing to problem #5 of assignment #5.]

Since $g \mid c$, there is some integer m such that $c = gm$.

But if $m \geq 1$, $c = gm \geq g$ - contradicting $c < g$.

Also, if $m \leq 0$, $c = gm \leq 0$ - contradicting $c > 0$.

Hence, no such c could exist.