5. Use induction to prove that for all positive integers n,

$$\prod_{i=1}^{n} \frac{i+2}{i} = \binom{n+2}{2}.$$

Basis Step: $n = 1$

$$\prod_{i=1}^{n} \frac{i+2}{i} = \frac{1+2}{1} = 3$$

$$\binom{n+2}{2} = \frac{3 \times 2}{2 \times 1} = 3$$

✓

Inductive Step:

Assume that $\prod_{i=1}^{n} \frac{i+2}{i} = \binom{n+2}{2}$

Prove that $\prod_{i=1}^{n+1} \frac{i+2}{i} = \binom{n+3}{2}$

$$\prod_{i=1}^{n+1} \frac{i+2}{i} = \left(\prod_{i=1}^{n} \frac{i+2}{i}\right) \cdot \left(\frac{n+3}{n+1}\right)$$

$$= \binom{n+2}{2} \cdot \left(\frac{n+3}{n+1}\right) \quad \text{(by induction hypothesis)}$$

$$= \frac{(n+2) \times (n+1)}{2 \times 1} \cdot \frac{n+3}{n+1}$$

$$= \frac{(n+3) \times (n+2)}{2 \times 1}$$

$$= \binom{n+3}{2}$$