CSE 321: Discrete Structures
Assignment #5
November 5, 2001
Due: Wednesday, November 14

Reading Assignment: Read Sections 4.3 - 4.5, 6.1.

Problems:

1. How many functions are there from the integers in the range [1,...,k] to the Boolean values 0, 1?
2. How many ways can three distinct numbers be chosen from $1, 2, \ldots, 100$ such that their sum is even?
3. Section 4.1, exercise 42.
4. Section 4.2, exercise 30.
5. An ice cream parlor has 28 different flavors, 8 different kinds of sauce, and 12 toppings.
   (a) In how many different ways can a dish of three scoops of ice cream be made where each flavor can be used more than once and the order of the scoops does not matter?
   (b) How many different kinds of small sundaes are there if a small sundae contains one scoop of ice cream, a sauce, and a topping?
   (c) How many different kinds of large sundaes are there if a large sundae contains three scoops of ice cream, where each flavor can be used more than once and the order of the scoops does not matter; two kinds of sauce, where each sauce can be used only once and the order of the sauces does not matter; and three toppings, where each topping can be used only once and the order ot toppings does not matter?
6. What is the coefficient of a^6b^6 in $(a^3 + b)^8$?
7. Prove the binomial theorem using mathematical induction.
8. Imagine a town with East-West streets numbered 1 through n, and North-South avenues numbered 1 through m. A taxi cab picks up a passenger at the corner of 1st street and 1st avenue. The passenger wishes to be delivered to n-th street and m-th avenue. It is quite clear that the passenger will be angry if the cab chooses a route longer than (n-1)+(m-1) blocks, so we won't allow the cabby to take a route longer than this. In other words, the cabby must always be increasing his street number or his avenue number. Suppose that there is an accident at i-th street and j-th avenue. How many routes can the cabby take that avoid the intersection with the accident?
9. A deck of 10 cards, each bearing a distinct number from 1 to 10, is shuffled to mix the cards thoroughly, so that each order is equally likely. What is the probability that the top three cards are in sorted (increasing) order?
10. A fair coin is flipped nn times. What is the probability that all the heads occur at the end of the sequence?