Discrete Structures

Counting

Chapter 4, Sections 4.1 - 4.3

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Permutations

◇ A permutation of a set of distinct objects is an ordered arrangement of these objects. An ordered arrangement of r elements of a set is called an r-permutation.

◇ **Theorem:** The number of r-permutations of a set with n distinct elements, where n is a positive integer and r is an integer with $0 \leq r \leq n$, is $P(n, r) = n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$

Combinations

◇ An r-combination of elements of a set is an unordered selection of r elements from the set.

◇ **Theorem:** The number of r-combinations of a set with n elements, where n is a positive integer and r is an integer with $0 \leq r \leq n$, is $C(n, r) = \frac{n!}{r!(n-r)!}$.

Examples

◇ In how many ways can ten adults and five children stand in a line so that no two children are next to each other?

◇ In how many ways can ten adults and five children stand in a circle so that no two children are next to each other?

◇ In how many ways can 20 students out of a class of 32 be chosen to attend class on a late Thursday afternoon (and take notes for the others) if

1. Paul refuses to go to class?
2. Michelle insists on going?
3. Jim and Michelle insist on going?
4. either Jim or Michelle (or both) go to class?
5. just one of Jim and Michelle attned?
6. Paul and Michelle refuse to attend class together?

Binomial Coefficients

◇ **Pascal's Identity:** Let n and k be positive integers with $n \geq k$. Then $C(n+1,k) = C(n,k-1) + C(n,k)$

◇ **Binomial Theorem:** Let x and y be variables, and let n be a positive integer. Then

$$\begin{aligned}(x+y)^n &= \sum_{j=0}^{n} C(n,j) x^{n-j} y^j \\ &= \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \ldots + \binom{n}{n-1} x y^{n-1} + \binom{n}{n} y^n\end{aligned}$$