

Midterm Practice Problems II

CSE 321

1. Answer the following questions with True or False:

- (a) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
- (b) $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
- (c) If A and B are sets, then $A - B = \overline{A \cap B}$.
- (d) If $a|b$ and $a|c$, then $b|c$.
- (e) If $a|(b + c)$, then $a|b$ and $a|c$.

Let $P(x, y)$ be the statement “ $x \geq y$ ”. Let the universe of x and y be the natural numbers (all integers greater than or equal to zero).

- (f) $\forall x \exists y P(x, y)$
- (g) $\exists x \forall y P(x, y)$
- (h) $\exists y \forall x P(x, y)$
- (i) $\neg \exists x \forall y \neg P(x, y)$

The function $y = x^2$ is:

- (j) one-to-one on the set of integers.
- (k) onto on the set of integers.
- (l) one-to-one on the set of **positive** integers.

2. Show using set builder notation that for sets A and B,

$$A \cup (B - A) = A \cup B.$$

3. Show that $4n + 3$ and $5n + 4$ are relatively prime. (Hint: Use Euclid’s algorithm. Do not use induction.)

4. Prove using induction that $n^2 - 7n + 12$ is nonnegative for $n \geq 3$.

5. Construct a logical argument using rules of inference to show that the following sentences imply the conclusion “It rained:”

- “If it does not rain or if it is not foggy, then the sailing race will be held and the life-saving demonstration will go on.”

- “If the sailing race is held, then the trophy will be awarded.”
- “The trophy was not awarded.”

Justify each step by indicating the rule you applied.