

Discrete Structures

Logic

Chapter 1, Sections 1.1–1.3

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Outline

- ◇ Propositional Logic
- ◇ Propositional Equivalences
- ◇ First-order Logic

Propositional Logic

Let p and q be propositions.

- ◇ **Negation** $\neg p$ The statement “It is not the case that p .” is true, whenever p is false and is false otherwise.
- ◇ **Conjunction** $p \wedge q$ The statement “ p and q ” is true when both p and q are true and is false otherwise.
- ◇ **Disjunction** $p \vee q$ The statement “ p or q ” is false when both p and q are false and is true otherwise.
- ◇ **Exclusive or** $p \oplus q$ The *exclusive or* of p and q is true when exactly one of p and q is true and is false otherwise.

Propositional Logic

Let p and q be propositions.

◇ **Implication** $p \rightarrow q$ The *implication* $p \rightarrow q$ is false when p is true and q is false and is true otherwise. p is called the **hypothesis** (antecedent, premise) and q is called the **conclusion** (consequence).

- “if p , then q ” “ p implies q ” “ p only if q ”
“ p is sufficient for q ” “ q is necessary for p ”
- $q \rightarrow p$ is called the **converse** of $p \rightarrow q$
- $\neg q \rightarrow \neg p$ is called the **contrapositive** of $p \rightarrow q$

◇ **Biconditional** $p \leftrightarrow q$ The *biconditional* $p \leftrightarrow q$ is true whenever p and q have the same truth values and is false otherwise.

Translating English Sentences

- ◇ You can access the Internet from campus only if you are a computer science major or you are not a freshman.

- ◇ You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.

Logical Equivalences

- ◇ **Tautology** A compound statement that is always true.
- ◇ **Contradiction** A compound statement that is always false.
- ◇ **Contingency** A compound statement that is neither a tautology nor a contradiction.
- ◇ **Logical equivalence** $p \Leftrightarrow q$ Propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology.

Tautologies?

◇ I don't jump off the Empire State Building implies if I jump off the Empire State Building then I float safely to the ground.

◇ $((\text{Smoke} \wedge \text{Heat}) \rightarrow \text{Fire}) \leftrightarrow ((\text{Smoke} \rightarrow \text{Fire}) \vee (\text{Heat} \rightarrow \text{Fire}))$

Logical Equivalences

$p \wedge T \Leftrightarrow p$ $p \vee F \Leftrightarrow p$	Identity laws
$p \vee T \Leftrightarrow T$ $p \wedge F \Leftrightarrow F$	Domination laws
$p \vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	Idempotent laws
$\neg(\neg p) \Leftrightarrow p$	Double negation law
$p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	Commutative laws
$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$ $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	De Morgan's laws

First-order Logic

- ◇ **Universal quantifier** \forall : The *universal quantification* of $P(x)$ is the proposition “ $P(x)$ is true for all values of x in the universe of discourse.”

- ◇ **Existential quantifier** \exists : The *existential quantification* of $P(x)$ is the proposition “There exists an element x in the universe of discourse such that $P(x)$ is true.”