CSE 321

Practice Problems for Final

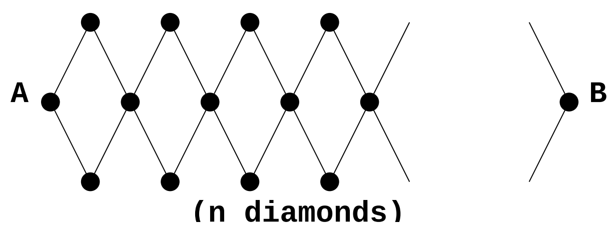
1. The formula $\neg(p \rightarrow \neg q) \rightarrow (r \rightarrow (\neg s \rightarrow t))$ is false for exactly one assignment to p, q, r, s and t. Find this assignment **without** constructing a truth table

2. Give logical expressions for the following statements. Use quantifiers, connectives, and the predicates P(x) and H(x) which mean "x passed the class" and "x turned in all of the homework".

   (a) Every student that passed the class turned in all of the homework.

   (b) There was a student that passed the class, but did not turn in all of the homework.

3. Prove that in the graph below, there are exactly 2^n paths of length $2n$ between vertex A and vertex B for $n \geq 1$.



4. Let G be a graph and define a relation R on the vertices of G s.t. $n_1 R n_2$ if there is a path between nodes n_1 and n_2 in G. Verify that R is an equivalence relation. How many equivalence classes does R have ?

5. Determine the number of strings over {a,b,c} of length 100 that have exactly 98 a's.

6. Give an example of a relation which is not reflexive, not symmetric, not anti-symmetric, and not transitive. (You are to give one relation that lacks all of these properties, not separate relations for each property.) Justify your answer.

7. Suppose R_1 and R_2 are transitive relations on a set A. Is the relation $R_1 \cup R_2$ necessarily a transitive relation? Justify your answer

8. For what values of m and n does the complete bipartite graph $K_{m,n}$ have a Hamilton circuit ?

9. Translate the following English sentences into predicate logic where the universe is the set of people and the allowable predicates are:

   - S(x) : x is a student
   - F(x,y) : x and y are friends
   - O(x,y) : x is older than y

   (a) Every student has a friend who is also a student.

   (b) There is someone who is older than all of his/her friends.

   (c) Write a predicate logic statement equivalent to the negation of each of the statements above that **does not use** negation anywhere except immediately in front of the predicate symbols S, F, and O.

10. Let n be a positive integer. A perfect matching on a set of 2n vertices is an undirected graph with n edges, such that each vertex has degree exactly 1. For example, there is one perfect matching on any set of two vertices (with edge set {{1,2}} if the vertices are {1,2}), and three distinct perfect matchings on four vertices (with edge sets {{1,2},{3,4}}, {{1,3},{2,4}}, and {{1,4},{2,3}} if the vertices are {1,2,3,4}). Prove by induction that the number of perfect matchings on 2n vertices is the product of the odd numbers less than 2n (so for n=2 it is 1x3).

11. Find predicates $P(x)$ and $Q(x)$ such that $\forall x(P(x) \rightarrow Q(x))$ is false, but $\forall x P(x) \rightarrow \forall x Q(x)$ is true.

12. If a student appears in a true/false test with ten questions, and he randomly guesses the answers, what is the probability that

    (a) he answers exactly five questions correctly.

    (b) he answers at least one question correctly.

13. Let the predicated D(x,y) mean "team x defeated team y" and P(x,y) mean "team x has played team y." Give quantified formulas with the following meanings:

(a) Every team has lost at least one game.

(b) There is a team that has beaten every team it has played.

14. 1. If D(x, y) is the predicate that x divides y, then which of the following statements are true in the domain of positive integers?

(a) $\forall x D(x, x)$

(b) $\forall x \forall y (D(x, y) \rightarrow D(y, x))$

(c) $\forall x \forall y ((D(x, y) \textit{ and } D(y, x)) \rightarrow (x = y))$

(d) $\forall x \forall y (D(x, y) \textit{ and } D(y, x))$

(e) $\forall x \forall y \forall z ((D(x, y) \textit{ and } D(y, z)) \rightarrow D(x, z))$

15. Prove the following for all natural numbers n by induction:

$$\sum_{i=0}^{n} \frac{i}{2^i} = 2 - \frac{n+2}{2^n}$$

16. The **complementary graph** $\overline{G}$ of a simple graph G has the same vertices as G. Two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. Find the following.

(a) $\overline{K_n}$

(b) $\overline{K_{m,n}}$

(c) $\overline{C_5}$