

CSE 321: Discrete Structures
Assignment #4
due: Wednesday, October 29

Reading: Rosen, Sections 3.1–3.3, 4.1–4.2

1. Let a , b and c be integers. Prove that if a does not divide bc , then a does not divide c .
2. Prove that any prime number bigger than 3 is congruent to 1 or 5 modulo 6.
3. Prove that $1 \cdot 1! + 2 \cdot 2! + \dots + n \cdot n! = (n + 1)! - 1$ whenever n is a positive integer.
4. Prove that $3^n < n!$ whenever n is a positive integer greater than 6.
5. Which amounts of money can be formed using just dimes and quarters? Prove your answer using a form of mathematical induction.
6. Use mathematical induction to prove that

$$\sum_{1 \leq k \leq n} k2^k = (n - 1)2^{n+1} + 2.$$

7. John and Sara have a party to which they invite n other married couples. As is normal at parties, handshaking took place. Of course, no one shook their own hand or their spouse's hand (and not everyone shook everyone else's hand). After all the handshaking was over, John asked all the other people present including his wife Sara "how many different people's hands did you shake this evening?" Interestingly, they each gave a different answer. From the information given, deduce how many different people's hands Sara shook that evening. Prove your answer by induction on n . (Hint: try working through the solution for several small values of n before going to the general case.)