CSE 321 Worksheet 321 Informal Solutions

Thursday, November 30, 2003

1. procedure mult(n:positive integer,x:integer)
   if *n=1* then *mult(n,x) := x*
   else *mult(n,x) := x + mult(n-1,x)*

2. If $n = 1$, then $n = nx$, and the algorithm correctly returns x. Assume that the algorithm correctly computes kx. To compute $(k + 1)x$ it recursively computes the product of $k + 1 - 1 = k$ and x, and then adds x. By the inductive hypothesis, it computes that product correctly, so the answer returned is $kx + x = (k + 1)x$, which is correct.

3. If $n = 0$, then the first line of the program correctly returns the function value 1. Assume that the program works correctly for $n = k$. If $n = k + 1$, then the else clause is executed, and the value a times power(a,k) is returned. Since the latter is $a^k$by inductive hypothesis, this equals $a * a^k = a^{k+1}$, which is correct.

4. $f\,(n) = n^2$Let P(n) be "$f\,(n) = n^2$."
   Basis step: P(1) is true since $f(1) = 1 = 1^2$, which follows form the definition of f.
   Inductive Step: Assume $f\,(n) = n^2$. Then $f(n+1) = f((n+1)-1) + 2(n+1) - 1 = f(n) + 2n + 1 = n^2 + 2n + 1 = (n+1)^2$.

5. Let P(n) be "$a + (a+d) + \ldots (a+nd) = \frac{(n+1)(2a+nd)}{2}$."
   Basis Step: P(1) is true since $a + (a+d) = 2a + d = \frac{2(2a+d)}{2}$.
   Inductive Step: Assume that P(k) is true. Then $a+(a+d)+\ldots(a+kd)+(a+(k+1)\,d) = \frac{(k+1)(2a+kd)}{2}+a+(k+1d) = \frac{1}{2}\left[2ak+2a+k^2d+2a+2kd+2d\right] = \frac{1}{2}\left[2ak+4a+k^2d+3kd+2d\right] = \frac{1}{2}\,(k+2)\,(2a+(k+1)d)$

6. *Basis Step:* When $n = 1$ there is one circle, and we can color the inside blue and the outside red to satisfy the conditions.
   Inductive Step: Assue the inductive hypothesis that if there are k circles, the the regions can be 2-colored such that no regions with a common boundary have the same color, and consider a situation with $k + 1$ circles. Remove one of the circles, producing a picture with k circles, and invoke the inductive hypothesis to color it in the prescribed manner. Then replace the removed circle and change the color of every region inside this circle. The resulting figure satisfies the condition, since if two regions have a common bounary, then either that boundary involved the new circle, in which case the regions on either side used to be the same region and not the inside portion is different from the outside, or else the boundary did not involve the new circle, in which case the regions are colored differently because they are colored differently before the new circle was restored.