

CSE 321: Discrete Structures  
Assignment #3  
April 18, 2003  
due: Friday, April 25

- For each of the following functions, state whether or not it is injective, and whether or not it is surjective. Justify your answers.
  - $f : \mathbf{N} \rightarrow \mathbf{N}$ , where  $f(n) = n^2$ .
  - $f : \mathbf{Z} \rightarrow \mathbf{N}$ , where  $f(n) = n^2$ .
  - $f : \mathbf{R} \rightarrow \mathbf{R}$ , where  $f(n) = 3n + 7$ .
  - $f : \mathbf{N} \rightarrow \mathbf{N}$ , where  $f(n) = \lceil n/3 \rceil$ .
  - $f : \mathbf{N} \rightarrow \mathbf{N}$ , where  $f(n) = 3 \lceil n/3 \rceil$ .
  - $f : \mathbf{N} \rightarrow \mathbf{N}$ , where  $f(n) = \begin{cases} n + 1, & \text{if } n \text{ is even} \\ n - 1, & \text{if } n \text{ is odd} \end{cases}$ .
- Suppose you graph a function  $f : \mathbf{R} \rightarrow \mathbf{R}$ . The fact that  $f$  is a function means that any straight vertical line will intersect the graph of  $f$  at exactly one point. What similar statement can you make about the graph of  $f$  if  $f$  is
  - injective?
  - surjective?
  - bijective?
- Let  $f : \mathbf{N} \rightarrow \mathbf{N}$  and  $g : \mathbf{N} \rightarrow \mathbf{N}$ , where  $f(x) = x \bmod 28$  and  $g(x) = x + 1$ . What are each of the functions  $f \circ g$  and  $g \circ f$ ? Either prove that these two functions are equal, or give a counterexample proving that they are unequal.
- Draw the graph of the function  $f : \mathbf{R} \rightarrow \mathbf{R}$ , where  $f(x) = \lfloor x/4 \rfloor$ . Be sure your graph extends into both positive and negative values of  $x$ .
- Section 2.3, exercise 4. Give a careful proof.
- Section 2.3, exercise 12. Justify your answer. The function  $n!$  is defined on page 85. (Hint: Think about the unique factorization of  $100!$  into primes. What about this factorization determines the number of zeros at the end of the decimal representation of  $100!$  ?)