CSE 321: Discrete Structures
Assignment #4
Due: Wednesday, April 27

**Reading Assignment:** Section 3.3- 3.4, 4.1-4.3 of Rosen.

**Problems:**

1. Use Euclid's algorithm to compute the following showing all the intermediate steps: $\gcd(3939, 143)$.

2. Let $a, b$ and $c$ be integers. Prove that if $a$ does not divide $bc$, then $a$ does not divide $c$.

3. (a) Let $a, b$ be positive integers. Define $S_{a,b}$ to be the set of all positive integers that can be written in the form $sa + tb$ for integers $s, t$. Prove that the smallest element in $S_{a,b}$ (why should it exist?) is in fact equal to $\gcd(a, b)$.

   (b) Prove that the linear equation $ax + by = c$ where $a, b, c$ are integers and $a \neq 0$ and $b \neq 0$ has a solution in integers $(x, y)$ if and only if $\gcd(a, b) | c$.

4. Section 3.3, exercise 10.

5. Section 3.3, exercise 12.

6. Section 3.3, exercise 44.

7. Use mathematical induction to prove that

$$\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2.$$