CSE 321: Discrete Structures

Solution to Practice Final:

1. True or False.
   (a) False
   (b) False
   (c) True
   (d) True
   (e) False
   (f) True
   (g) True
   (h) False
   (i) True
2. Fill in the Blanks
   (a) B
   (b) A
   (c) 2^n
   (d) $\binom{r}{k}$
   (e) $\binom{100}{10} \cdot 5^{10}$
   (f) 2^{mn}
   (g) $2^{2 \cdot \binom{n}{2}}$
   (h) $2^{\binom{n}{2}}$
   (i) $\binom{21}{7} \cdot 10^7 \cdot 2^{14}$
   (j) $\binom{25}{12} \cdot (\frac{4}{5})^{12} \cdot (\frac{1}{5})^{13}$
3. (a) $v_n = 2v_{n-1} + 1$

4. (a) 2^{25}

   (b) $1/2$

   (c) $(1/4)/(3/4) = 1/3$

   (d) $25 \cdot 1/2 = 12.5$

   (e) $\sum_{i=1}^{25}(2 \cdot 1/2 - 1 \cdot 1/2) = 12.5$

5. (a) $8\,!$

   (b) $8\,! - 6\,!$

   (c) $8\,! - 6\,! - 5\,! + 3\,!$

   (d) $\binom{10}{3} \cdot 7^7$

6. (a) $\frac{(3/4)^{50} \cdot (1/4)^{50}}{\binom{100}{50}(3/4)^{50} \cdot (1/4)^{50}} = \frac{1}{\binom{100}{50}}$

   (b) $100 \cdot 3/4 = 75$

   (c) The expected return is $50 \cdot [(3/4)^2 + (1/4)^2] + 100 \cdot [2 \cdot (3/4) \cdot (1/4)]$.

7. $486 = 1 \cdot 446 + 40$

   $446 = 11 \cdot 40 + 6$

   $40 = 6 \cdot 6 + 4$

   $6 = 1 \cdot 4 + 2$

   $4 = 2 \cdot 2 + 0$

   Thus, gcd $(486, 446) = 2$

8. (a) $\frac{\binom{100}{7} \cdot \binom{n-100}{93}}{\binom{n}{100}}$

   (b) $\binom{100}{7} \cdot (\frac{100}{n})^7 \cdot (1 - \frac{100}{n})^{93}$

9. By induction on the number of vertices. (Note that in an undirected, connected graph without cycles, there must exist a vertex with degree one).

10.

11. (a) The reflexive-symmetric-transitive closure of R is $\{(1,2),(1,3),(2,4),(5,6),(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(2,1),(3,1),(4,2),(6,5),(1,4),(4,1)\}$

    (b) $2^{\binom{6}{2}}$

12. (a) 6^{26}

    (b) $6^{26} - \binom{26}{6} \cdot 6\,!$

    (c) R is not an equivalence relation, since it's not reflexive.

    (d) It's an equivalence relation. And there are $\binom{6}{3}\binom{3}{2}$ words.