Bayes Formula

$$p(A \wedge B) = p(A \mid B)\,p(B) = p(B \mid A)\,p(A)$$

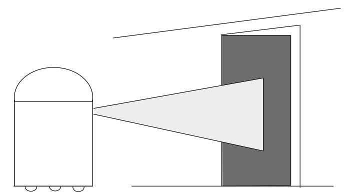
$$\Rightarrow$$

$$p(A \mid B) = \frac{p(B \mid A)\; p(A)}{p(B)}$$

1

A Simple Example: State Estimation

- Suppose a robot obtains measurement *s*
- What is *p(doorOpen|s)?*



2

Causal vs. Diagnostic Reasoning

- *p(open|s) is diagnostic*
- *Often causal knowledge like*

$$p(s \mid open)$$

count frequencies!

is easier to obtain.

- *Application of Bayes rule:*

$$p(open \mid s) = \frac{p(s \mid open)\,p(open)}{p(s)}$$

3

Normalization

$$p(open \mid s) = \frac{p(s \mid open)\,p(open)}{p(s)}$$

$$p(s) = p(s \wedge open) + p(s \wedge \neg open)$$

$$\Rightarrow$$

$$p(s) = p(s \mid open)\,p(open) + p(s \mid \neg open)\,p(\neg open)$$

$$\Rightarrow$$

$$p(open \mid s) = \frac{p(s \mid open)\,p(open)}{p(s \mid open)\,p(open) + p(s \mid \neg open)\,p(\neg open)}$$

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Example

- $p(s|open) = 0.6$ $\quad$ $p(s|\neg open) = 0.3$
- $p(open) = p(\neg open) = 0.5$

$$p(open \mid s) = \frac{p(s \mid open)\,p(open)}{p(s \mid open)\,p(open) + p(s \mid \neg open)\,p(\neg open)}$$

$$p(open \mid s) = \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = \frac{2}{3} = 0.67$$

Observing *s* raises the probability that the door is open.

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Integrating a second Measurement …

- New measurement s_2
- $p(s_2|open) = 0.5$ $\quad$ $p(s_2|\neg open) = 0.6$

$$p(open \mid s_2 s_1) = \frac{p(s_2 \mid open)\,p(open \mid s_1)}{p(s_2 \mid open)\,p(open \mid s_1) + p(s_2 \mid \neg open)\,p(\neg open \mid s_1)}$$

$$p(open \mid s_2 s_1) = \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{1}{2} \cdot \frac{2}{3} + \frac{3}{5} \cdot \frac{1}{3}} = \frac{5}{8} = 0.625$$

s_2 lowers the probability, that the door is open.

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Another Example: Rare Disease

- *Disease* d with $p(d) = 0.0001$ and $p(\neg d) = 0.9999$
- Test t with $p(t|d) = 0.9$ and $p(t|\neg d) = 0.01$

$$p(d\mid t) = \frac{p(t\mid d)\,p(d)}{p(t\mid d)\,p(d) + p(t\mid \neg d)\,p(\neg d)}$$

$$p(d\mid t) = \frac{0.9 * 0.0001}{0.9 * 0.0001 + 0.01 * 0.9999} = \frac{0.00009}{0.010089} < \frac{9}{1000}$$

Even though the test seems very good and has few false positives, the probability of having the disease given a positive test is very small

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