CSE 321: Discrete Structures | Assignment #5
November 6, 2006 | Due: Monday, November 13

Reading Assignment: Rosen's text 6th Edition: Finish sections 2.1-2.3 and read 5.1-5.4 (or, 5th Edition: sections 1.6-1.8, 4.1-4.4).

Problems:

1. Which of the following statements is true?

   (a) $\{x\} \subseteq \{x\}$
   (b) $\{x\} \in \{x, \{x\}\}$
   (c) $\{x\} \in \{x\}$
   (d) $\{x, \{x\}\} \subseteq \mathcal{P}(\{x\})$
   (e) $\emptyset \in \emptyset$
   (f) $\emptyset \in \mathcal{P}(\emptyset)$
   (g) $\emptyset \subseteq \{x\}$
   (h) $\{\emptyset\} \subseteq \emptyset$

2. Can you conclude that $A = B$ if A, B, C are sets such that

   (a) $A \cup C = B \cup C$
   (b) $A \cap C = B \cap C$
   (c) $A \cup B = B \cup C$ and $A \cap C = B \cap C$

   Justify your answers.

3. 6th edition, Section 4.3, Problem 22.
   5th edition, Section 3.4, Problem 22.
   (Note that this asks that you show equality between two sets.)

4. If Σ is an alphabet, for $x \in \Sigma^*$ we define the *reversal of* x recursively as follows:

   - **Basis:** $\lambda^R = \lambda$ where λ is the empty string
   - **Recursive step:** $(ua)^R = au^R$ for $a \in \Sigma$, $u \in \Sigma^*$

   Show using structural induction on $x \in \Sigma^*$ that if $w, x \in \Sigma^*$ are two strings in Σ^* then

   $$(wx)^R = x^R w^R.$$

5. Solve the following counting problems. In each case, show the reasoning that leads you to your answer.

(a) How many different truth tables are there for propositions in n variables?

(b) How many bit strings of length 10 contain either five consecutive 0's or five consecutive 1's?

(c) Suppose that you have n beads, each of a different color, that you need to string into a circular necklace. How many distinct necklaces can you make?

6. 6th edition, Section 5.1, Problem 22.
   5th edition, Section 4.1, Problem 20.

7. In the town of Smallsville there are three elected positions: mayor, sheriff, and dog-catcher, to be chosen from among the 78 people in town. Each person can be elected to at most one of these positions.

   (a) How many ways are there of filling all three of these positions? That is, how many different election outcomes are possible (assuming no ties)?

   (b) During the election four years ago there were only 67 people in town and someone forgot to prohibit the same person from being elected to more than one position. How many outcomes were possible then?

   (c) After a recent scandal involving a terrier, it was decided that, for the next election, D.R. Rotweiler, one of the townspeople in Smallsville, will no longer eligible to be chosen as dog-catcher. In how many ways can the three positions be chosen next time?

8. In a dinner party with n people, all of them are seated at a circular table. Suppose there is a name tag at each place of the table, and suppose that nobody sits down in their correct place. Show that it is possible to rotate the table so that at least two people are sitting in the correct place.

9. **Extra Credit** Assume that friendship is always mutual; that is, if A is a friend of B then B is also a friend of A. Show that under this assumption in any group of people there are always two people who have exactly the same number of friends within the group.

10. **Extra Credit** Let A, B be two subsets of $\{1, 2, \ldots, m\}$ of size n each, where $n > \sqrt{2m}$. Prove that there must exist pairs $(a, b) \in A \times B$ and $(a', b') \in A' \times B'$ with $a \neq a'$ and $b \neq b'$ such that $a + b = a' + b'$.