CSE 321: Discrete Structures
Assignment #2
Due: Friday, April 14

**Reading Assignment:** Rosen, 5th edition: Sections 1.3-1.8 pp. 233-236, 2.4-2.5.

**Problems:**

1. Let $Q(x, y)$ be the statement "$x$ has been a contestant on $y$". Express the following sentences in terms of $Q(x, y)$, quantifiers and logical connectives, where the universe of discourse for $x$ is the set of all students at your school and the universe of discourse for $y$ is the set of all quiz shows on television. Then give the negation of the statement in English.
   - No student at your school has ever been a contestant on a television quiz show.
   - Every television quiz show has had a student from your school as a contestant.
2. Prove or disprove the claim that $\forall x(P(x) \rightarrow Q(x))$ is logically equivalent to $\forall x P(x) \rightarrow \forall x Q(x)$.
3. Let $Q(A, B)$ be the statement $A \subseteq B$. If the universe of discourse for both $A$ and $B$ is all sets of integers, what are the truth values of the following? Justify your answers.
   - $(\forall A)(\exists B)Q(A, B)$
   - $(\forall B)(\exists A)Q(A, B)$
   - $(\exists A)(\forall B)Q(A, B)$
   - $(\forall A)(\forall B)Q(A, B)$
4. Section 1.5, Exercise 12.
5. Section 1.5, Exercise 22.

6. Prove that if you pick 10 numbers from 1 to 1000, there is a pair of numbers such that the larger of the two is at most twice the other.

7. Which of the following statements are true?

   - $\{x\} \subseteq \{x\}$
   - $\{x\} \in \{x, \{x\}\}$
   - $\{x\} \in \{x\}$
   - $\{x, \{x\}\} \subseteq \mathcal{P}(\{x\})$

8. Carefully prove the following implications.

   - $(A \cup B = B) \rightarrow (A \subseteq B)$
   - $(A \cap B = A) \rightarrow (A \subseteq B)$

9. Give an example of a function from $\mathcal{N}$ to $\mathcal{N}$ which is

   - one-to-one but not onto
   - onto but not one-to-one
   - both onto and one-to-one (but different from the identity function)
   - neither one-to-one nor onto.