CSE 321: Discrete Structures Spring 2006

PROBLEM SET 5

Due Friday, May 12, 2006, in class

Reading: Sections 4.1– 4.4, Chapter 5.

1. Solve the following counting problems. In each case show the reasoning that leads you to your answer.

   (a) A palindrome is a word that reads the same forwards and backwards. How many seven-letter palindromes can be made from the English alphabet?

   (b) Suppose you have n beads, each of a different color, that you need to string into a necklace? How many distinct necklaces can you make? (A necklace flipped over remains the same and does not count as a distinct necklace.)

   (c) How many different truth tables are there for propositions in n variables?

   (d) How many 5 card hands from a 52 card deck have the same number of Diamonds and Hearts?

   (e) How many bit strings of length 10 contain either five consecutive 0's or five consecutive 1's?

2. Imagine a town with East-West streets numbered 1 through n and North-South Avenues numbered 1 through m. A taxi cab picks you up at the corner of 1st Avenue and 1st street, and you wish to be dropped off at the corner of m'th avenue and n'th street. Since you are a smart 321 student, you are obviously not going to permit the cab driver to drive longer than the necessary $(n-1)+(m-1)$ blocks. In other words, the cabby must always be increasing his street number or his avenue number. Suppose there is an accident at the intersection of street i and avenue j, for some i, j where $1 < i < n$ and $1 < j < m$. How many routes can the cab driver take to get you to your destination while avoiding the intersection with the accident? Justify your answer.

3. (a) In a dinner party with n people, all of them are seated at a circular table. Suppose there is a nametag at each place at the table and suppose that nobody sits down in their correct place. Show that it is possible to rotate the table so that at least two people are sitting in the correct place.

   (b) Prove that every set of ten distinct numbers between 1 and 100 contains two disjoint nonempty subsets with the same sum.

4. (a) Section 4.3, Exercise 26.

   (b) Section 4.3, Exercise 28.

5. Section 4.4, Exercise 22.

6. Prove the Binomial Theorem using mathematical induction.