

PROBLEM SET 8  
Due Friday, June 2, 2006, in class

**Reading:** Sections 8.1–8.5,8.7

1. For the relation  $R = \{(b, c), (b, e), (c, e), (d, a), (e, b), (e, c)\}$  on  $\{a, b, c, d, e\}$ , draw the following relations in directed graph form:
  - (a) The reflexive closure of  $R$ .
  - (b) The symmetric closure of  $R$ .
  - (c) The transitive closure of  $R$ .
  - (d) The reflexive, symmetric, transitive closure of  $R$ .
2. Let  $R$  be the relation on the set of ordered pairs of positive integers such that  $((a, b), (c, d)) \in R$  if and only if  $ad = bc$ . Show that  $R$  is an equivalence relation. (Can you identify what familiar objects the equivalence classes correspond to?)
3. Prove that any (simple, undirected) graph on  $n \geq 2$  vertices contains two vertices of equal degree.
4. Prove that if  $G$  is disconnected, then  $\overline{G}$ , the complement of  $G$ , is connected. (Recall that  $\overline{G}$  contains all and only those edges that are absent in  $G$ .)
5. Section 8.2, Exercise 36. (See example 11.)
6. Section 8.3, Exercise 40. (You'll need to read section 8.3 for the definitions.)
7. Section 8.5, Exercise 26. (You'll need to read the sections for the definitions of these graphs.)