

**Problems:**

1. Section 1.1, exercise 10
2. Section 1.1, exercise 18, parts (a) to (e)
3. State in English the converse and contrapositive of each of the following implications:
  - (a) If  $a$  is pushed onto the stack before  $b$ , then  $b$  is popped before  $a$ .
  - (b) If the input is correct and the program terminates, then the output is correct. (Be sure to use De Morgan's Law to simplify the contrapositive.)
4. Section 1.1, exercise 58
5. The following two statements form the basis of the most important methods for automated theorem proving. Use truth tables to prove that they are tautologies.
  - (a) Resolution:  $((p \vee q) \wedge (\neg q \vee r)) \rightarrow (p \vee r)$
  - (b) Modus ponens:  $((p \wedge (p \rightarrow q)) \rightarrow q)$
6. Show that Modus ponens is a tautology without using a truth table. Show each step and indicate which logical equivalences you use.
7. Section 1.2, exercise 22
8. Give the negation of each of the following statements:
  - All good students study hard.
  - No students in mathematics are unable to use a computer.
  - $\forall x \exists y x = y^2$
9. Prove or disprove the claim that the compound proposition  $\forall x P(x) \vee \forall x Q(x)$  is logically equivalent to  $\forall x (P(x) \vee Q(x))$
10. Section 1.3, exercise 62
11. Prove or disprove the claim that the proposition  $\forall x (P(x) \rightarrow Q(x))$  is logically equivalent to  $\forall x P(x) \rightarrow \forall x Q(x)$

**(more on next page)**

12. Section 1.4, exercise 8

13. Determine the truth value of  $\exists x \forall y (x \leq y^2)$  when the universe of discourse is the

- (a) Positive reals
- (b) Non-negative reals
- (c) Positive integers
- (d) Non-negative integers

14. Section 1.4, exercise 52

Please write how many hours (to the nearest quarter-hour) it took you to complete this assignment near where you write your name on the first page.