University of Washington
Department of Computer Science and Engineering
CSE 321, Winter 2008

February 12, 2008

Homework 6, Due Wednesday, February 20, 2008

Problem 1:

Let f_n be the n-th Fibonacci number. Prove $f_1 + f_3 + \cdots + f_{2n-1} = f_{2n}$ whenever n is a positive integer.

Problem 2:

Let f_n be the n-th Fibonacci number and suppose $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

Show that $A^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}$ when n is a positive integer.

Problem 3:

Give a recursive definition of

a) the set of odd positive integers,

b) the set of positive integers congruent to 1 modulo 5 or conguent to 3 modulo 5.

c) the set of positive integers whose only prime factors are 2 and/or 5.

Problem 4:

Let w be a bit string that starts with 0. Prove that if w ends with 0, the string 01 occurs in w the same number of times as the string 10, and if w ends in 1, the string 01 occurs in w one more time than the string 10. Hints: give a recursive definition of W, the set of bit strings that start with 0 and then prove by structural induction that the above property holds.

Problem 5:

Give a recursive definition for the set of palindromes over $\Sigma = \{a, b\}$. A palindrome is a string that is the same as its reversal.

Problem 6:

A full binary tree is defined with the following rules.

Basis step: A singleton node r is a full binary tree.

Recursive step: If T_1 and T_2 are full binary trees with roots r_1 and r_2 respectively and r is a node, then a full binary tree T with root r can be created by connecting r to r_1 as the left subtree and connecting r to r_2 as the right subtree.

A node is a leaf if it has no children, and is an internal node otherwise. Prove by structural induction that the number of leaves is one greater than the number of internal nodes.

Extra Credit 7:

Give a recursive definition for the set of strings over $\Sigma = \{a, b\}$ that have more a's than b's. (One way to solve this problem is to first give a recursive definition of the set of strings with the same number of a's as b's.) Give an argument that your definition is correct.