CSE 321 Discrete Structures

Winter 2008
Lecture 2
Propositional Equivalences

Announcements

- Homework 1, Due January 16th
- Reading: sections 1.1, 1.2, 1.3
- Quiz section Thursday
  - 12:30-1:20 or 1:30 – 2:20
  - CSE 305
- Office hours
  - Richard Anderson, CSE 582, Friday 2:30-3:30
  - Natalie Linnell, CSE 218, Monday, 11:00-12:00, Tuesday, 2:00-3:00

Highlights from Lecture 1

- Fundamental tasks in computing
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  -
- Propositional logic
  - Proposition: statement with a truth value
  - Basic connectives
    - $\neg, \vee, \wedge, \rightarrow, \oplus, \leftrightarrow$
  - Truth table for implication

p	q	$p \rightarrow q$

Biconditional $p \leftrightarrow q$

- p iff q
- p is equivalent to q
- p implies q and q implies p

p	q	$p \leftrightarrow q$

English and Logic

- You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old
  - q: you can ride the roller coaster
  - r: you are under 4 feet tall
  - s: you are older than 16

Logical equivalence

- Terminology: A compound proposition is a
  - Tautology if it is always true
  - Contradiction if it is always false
  - Contingency if it can be either true or false

$p \vee \neg p$

$(p \oplus p) \vee p$

$p \oplus \neg p \oplus q \oplus \neg q$

$(p \rightarrow q) \wedge p$

$(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

Logical Equivalence

- *p* and *q* are *Logically Equivalent* if $p \leftrightarrow q$ is a tautology.
- The notation $p \equiv q$ denotes *p* and *q* are logically equivalent
- Example: $(p \rightarrow q) \equiv (\neg p \vee q)$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$	$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$

Computing equivalence

- Describe an algorithm for computing if two logical expressions are equivalent
- What is the run time of the algorithm?

Understanding connectives

- Reflect basic rules of reasoning and logic
- Allow manipulation of logical formulas
  - Simplification
  - Testing for equivalence
- Applications
  - Query optimization
  - Search optimization and caching
  - Artificial Intelligence
  - Program verification

Properties of logical connectives

- Identity
- Domination
- Idempotent
- Commutative
- Associative
- Distributive
- Absorption
- Negation

De Morgan's Laws

- $\neg (p \vee q) \equiv \neg p \wedge \neg q$
- $\neg (p \wedge q) \equiv \neg p \vee \neg q$

- What are the negations of:
  - Casey has a laptop and Jena has an iPod
  - Clinton will win Iowa or New Hampshire

Equivalences relating to implication

- $p \rightarrow q \equiv \neg p \vee q$
- $p \rightarrow q \equiv \neg q \rightarrow \neg p$
- $p \vee q \equiv \neg p \rightarrow q$
- $p \wedge q \equiv \neg (p \rightarrow \neg q)$
- $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
- $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
- $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
- $\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Logical Proofs

- To show P is equivalent to Q
  - Apply a series of logical equivalences to subexpressions to convert P to Q
- To show P is a tautology
  - Apply a series of logical equivalences to subexpressions to convert P to **T**

Show $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

Show $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not equivalent