CSE 321 Discrete Structures

Winter 2008
Lecture 3
Propositional Proofs

Announcements

- Reading for Monday: 1.3
- Signup for the mailing list
  - See course website for details
- Office hours
  - Richard Anderson, CSE 582, Friday 2:30-3:30
  - Natalie Linnell, CSE 218, Monday, 11:00-12:00, Tuesday, 2:00-3:00

Highlights from Lecture 2

- Tautology: A compound proposition that is always true.
- P is equivalent to Q ($P \equiv Q$): $P \leftrightarrow Q$ is a tautology
- Truth table algorithm for testing equivalence
- DeMorgan's laws
  - $\neg (p \vee q) \equiv$ ___________
  - $\neg (p \wedge q) \equiv$ ___________

Equivalences relating to implication

- $p \rightarrow q \equiv \neg p \vee q$
- $p \rightarrow q \equiv \neg q \rightarrow \neg p$
- $p \vee q \equiv \neg p \rightarrow q$
- $p \wedge q \equiv \neg (p \rightarrow \neg q)$
- $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
- $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
- $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
- $\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Logical Proofs

- To show P is equivalent to Q
  - Apply a series of logical equivalences to subexpressions to convert P to Q
- To show P is a tautology
  - Apply a series of logical equivalences to subexpressions to convert P to **T**

Why bother with logical proofs when we have truth tables?

Show $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

Show $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not equivalent

Predicate Calculus

- *Predicate* or *Propositional Function*
  - A function that returns a truth value
- "*x* is a cat"
- "*x* is prime"
- "student *x* has taken course *y*"
- "$x > y$"
- "$x + y = z$"

Quantifiers

- $\forall x\, P(x)$: $P(x)$ is true for every x in the domain
- $\exists x\, P(x)$: There is an x in the domain for which $P(x)$ is true

Statements with quantifiers

Domain: Positive Integers

Even(*x*)
Odd(*x*)
Prime(*x*)
Greater(*x*,*y*)
Equal(*x*,*y*)

- $\exists x$ Even(x)
- $\forall x$ Odd(x)
- $\forall x$ (Even(x) $\vee$ Odd(x))
- $\exists x$ (Even(x) $\wedge$ Odd(x))
- $\forall x$ Greater(x+1, x)
- $\exists x$ (Even(x) $\wedge$ Prime(x))

Statements with quantifiers

Domain: Positive Integers

Even(*x*)
Odd(*x*)
Prime(*x*)
Greater(*x*,*y*)
Equal(*x*,*y*)

- $\forall x \exists y$ Greater (y, x)
- $\forall x \exists y$ Greater (x, y)
- $\forall x \exists y$ (Greater(y, x) $\wedge$ Prime(y))
- $\forall x$ (Prime(x) $\rightarrow$ (Equal(x, 2) $\vee$ Odd(x))
- $\exists x \exists y$(Equal(x, y + 2) $\wedge$ Prime(x) $\wedge$ Prime(y))

Statements with quantifiers

Domain:
Positive Integers

Even(*x*)
Odd(*x*)
Prime(*x*)
Greater(*x*,*y*)
Equal(*x*,*y*)

- "There is an odd prime"
- "If x is greater than two, x is not an even prime"
- $\forall x \forall y \forall z$ ((Equal(z, x+y) $\wedge$ Odd(x) $\wedge$ Odd(y))$\rightarrow$ Even(z))
- "There exists an odd integer that is the sum of two primes"

Goldbach's Conjecture

- Every even integer greater than two can be expressed as the sum of two primes

Even(*x*)
Odd(*x*)
Prime(*x*)
Greater(*x*,*y*)
Equal(*x*,*y*)

Domain:
Positive Integers