CSE 321 Discrete Structures

Winter 2008
Lecture 12
Induction

Announcements

- Readings
  - Monday, Wednesday
    - Induction and recursion
      - 4.1-4.3 (5th Edition: 3.3-3.4)
  - Midterm:
    - Friday, February 8
    - In class, closed book
    - Estimated grading weight:
      - MT 12.5%, HW 50%, Final 37.5%

Highlights from Lecture 11

- Public Key Cryptography

Induction Example

- Prove $3 \mid 2^{2n} - 1$ for $n \geq 0$

Induction as a rule of Inference

$$\frac{P(0) \quad \forall k\,(P(k) \rightarrow P(k+1))}{\therefore \forall n\, P(n)}$$

$1 + 2 + 4 + \ldots + 2^n = 2^{n+1} - 1$

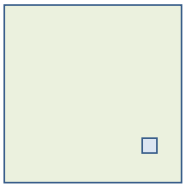
Harmonic Numbers

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \frac{1}{n} = \sum_{k=1}^{n} \frac{1}{k}$$

Prove $H_{2^n} \geq 1 + \frac{n}{2}$

Cute Application: Checkerboard Tiling with Trinominos

Prove that a $2^k \times 2^k$ checkerboard with one square removed can be tiled with: 



Strong Induction

P(0)
$\forall$ k ((P(0) $\wedge$ P(1) $\wedge$ P(2) $\wedge$ … $\wedge$ P(k)) $\rightarrow$ P(k+1))
$\therefore \forall$ n P(n)

Player 1 wins n $\times$ 2 Chump!

Winning strategy: chose the lower corner square

Theorem: Player 2 loses when faced with an n $\times$ 2 board missing the lower corner square